MATHEMATICS

Part - 1

Standard
VII

Government of Kerala
Department of General Education

Prepared by
State Council of Educational Research and Training (SCERT) Kerala

2024

ST-359-1-MATHS (E)-7-VOL-1

THE NATIONAL ANTHEM

Jana-gana-mana adhinayaka, jaya he
Bharatha-bhagya-vidhata
Punjab-Sindh-Gujarat-Maratha
Dravida-Utkala-Banga
Vindhya-Himachala-Yamuna-Ganga
Uchchala-Jaladhi-taranga
Tava subha name jage,
Tava subha asisa mage,
Gahe tava jaya gatha
Jana-gana-mangala-dayaka jaya he
Bharatha-bhagya-vidhata
Jaya he, jaya he, jaya he,
Jaya jaya jaya, jaya he.

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give my parents, teachers and all elders, respect and treat everyone with courtesy.

To my country and my people, I pledge my devotion. In their well-being and prosperity alone, lies my happiness.

MATHEMATICS

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Typesetting and Layout : SCERT
First Edition : 2024
Printed at : KBPS, Kakkanad, Kochi-30

Dear children,

We've acquired some of the basic concepts of mathematics.

Such as counting numbers, fractions, decimal forms and some algebra too. We've also had ample opportunities to use these to solve problems logically, explain them in terms of cause and effect, and to complete patterns.

We are moving ahead. To more applications of the concepts learnt, to more computational techniques, to develop more skill in recognising relations between numbers, explaining them in our own language and in the language of mathematics, to more deeper analysis of geometry, to more complex problem solving, to more possibilities of mathematical and computational thinking.

Let's march ahead together with confidence, thinking, enquiring and enjoying ourselves.

With love and regards,

Dr. Jayaprakash R.K.
Director
SCERT, Kerala

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CONTENTS

Certain icons are used in this textbook for convenience


Let's do problems


ICT possibilities

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a [1]**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the [2][unity and integrity of the Nation];

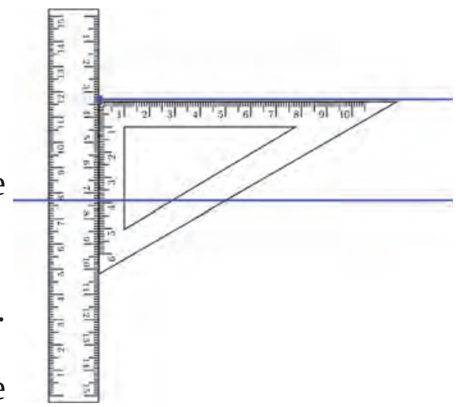
IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

---

1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec.2, for "Unity of the Nation" (w.e.f. 3.1.1977)

1 Parallel Lines

Let us recall

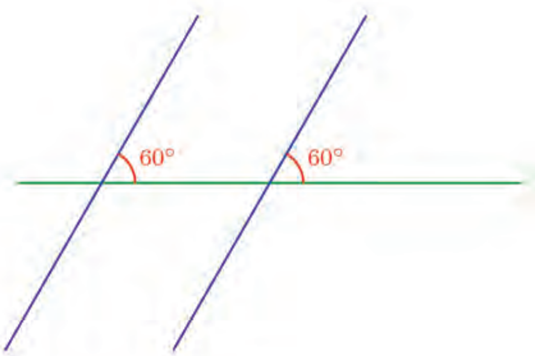


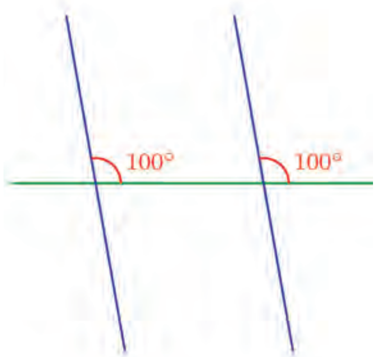
We have heard about parallel lines in class 6.

Lines that don't meet, keeping the same distance between them.

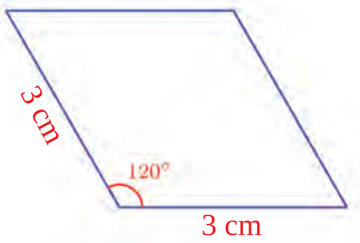
We have also drawn them with a scale and a set square.

Two lines drawn at the same slant to a given line are parallel. We've seen this also.



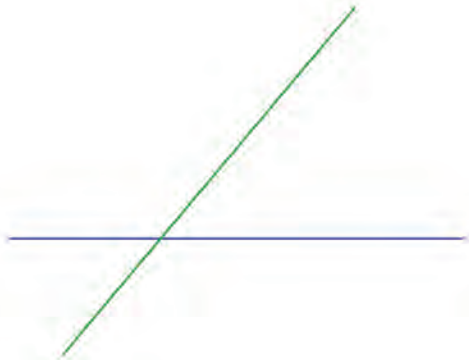


We know that a parallelogram is a quadrilateral in which the two pairs of opposite sides are parallel. Can you draw this parallelogram with measures as given ?

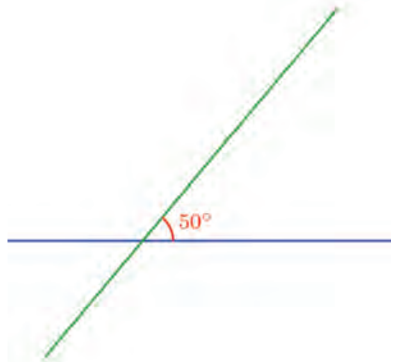


Lines and angles

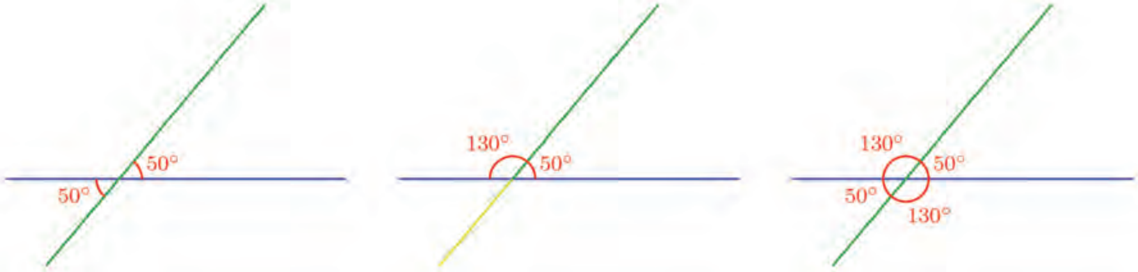
When a line crosses another line, how many angles are formed between them ?



If we know one of these, can we calculate the others ?

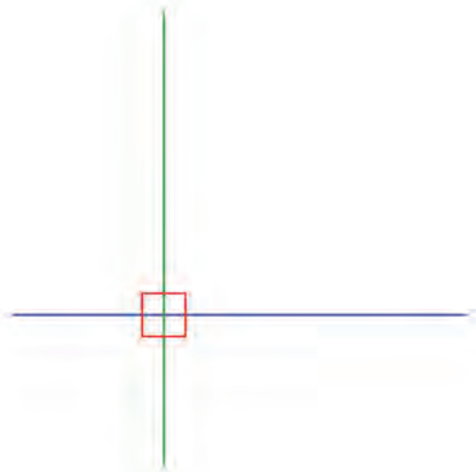


This too was seen in class 6.

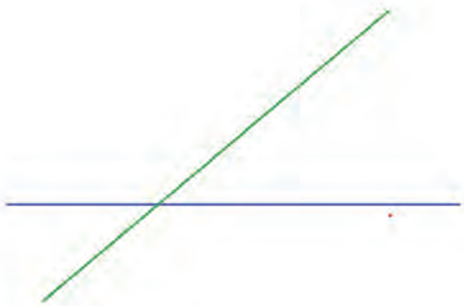


What can we say about the relation between four such angles in general ?

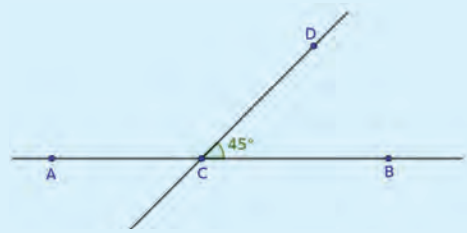
Nothing much to say if the crossing lines are perpendicular. All angles are 90°.



What if one line is a bit tilted ?

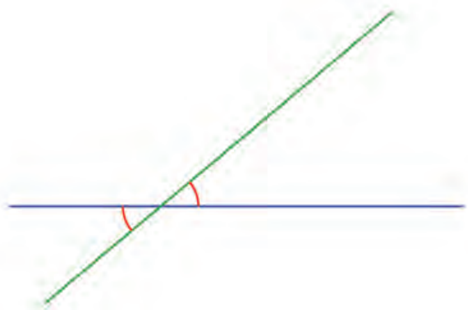


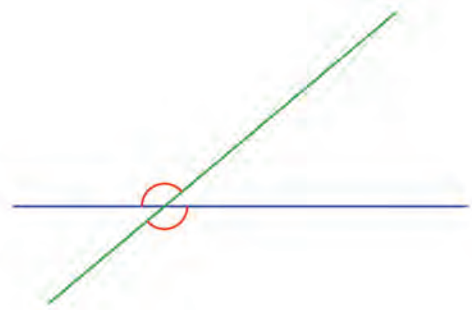
Draw a line AB. Mark a point C on the line and another point D outside the line. Draw a line through C and D. Find $\angle BCD$. Use the **Angle** *tool* and click on B, C, D in order (see what happens if you click in a different order).



Mark the other angles also like this. What is the relation between the angle measures ? Try changing the position of D. Don't you see a change in the angles ? Does the relation change ?

Two small angles and two large angles.

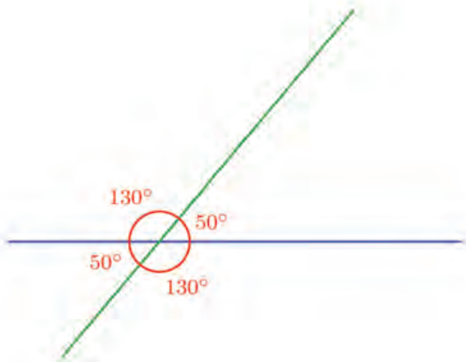




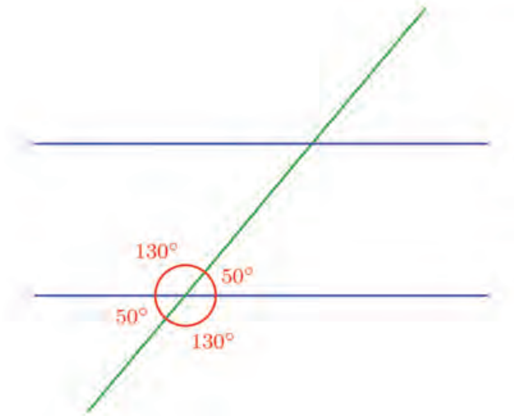
What is the relation between them?

- The two small angles are of the same measure.
- The two large angles are of the same measure.
- The sum of a small angle and a large angle is 180°.

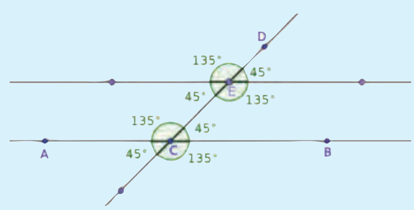
Look at the earlier figure once again:



What if we draw another line above, parallel to the blue line ?



Draw two intersecting lines and mark the angles between them as before. Mark a point *E* on *CD* and draw a line through it parallel to *AB*. Mark the four angles around *E* (You may mark more points on the line for ease of marking the angles).

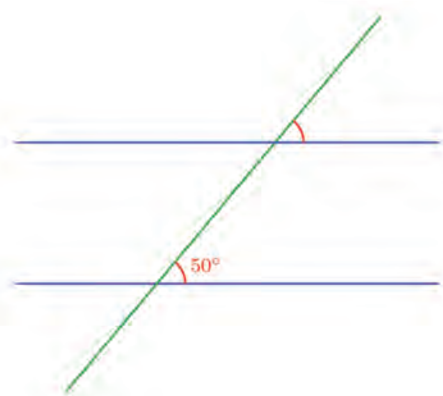


What is the relation between the eight angles you have marked now ? Try changing the position of *D*. Does the relation change when the angle measures change ? Try changing the position of *E*.

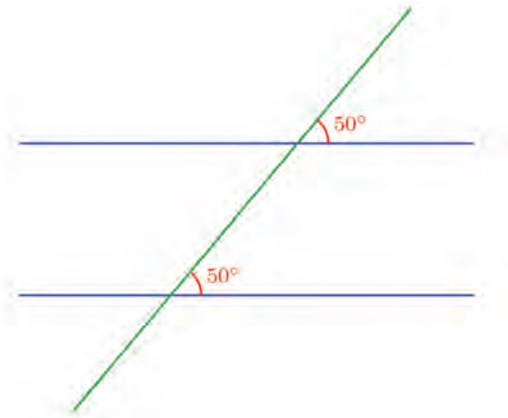
What happens when *E* takes the place of *C* ?

Now the blue line above makes four angles with the green line. What can you say about them?

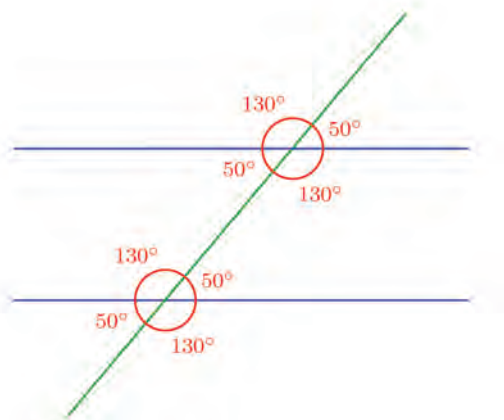
Let's look only at the small angle below and the marked angle above.



The blue lines are parallel. So these two must be of the same measure.



What about the other angles above ?

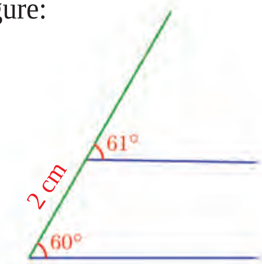


Suppose we start with an angle other than 50°. The measures of the other angles will change. But the relation between the angles will be the same. That is,

A line intersects two parallel lines at angles of the same measure.

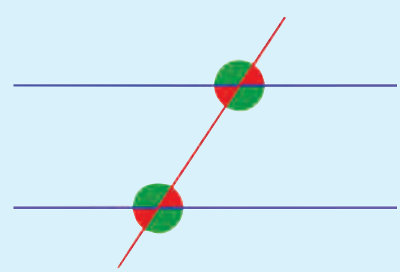
A change in angle

Suppose there is slight change in the slant of two lines with another line. The two lines won't be parallel. For example, look at this figure:

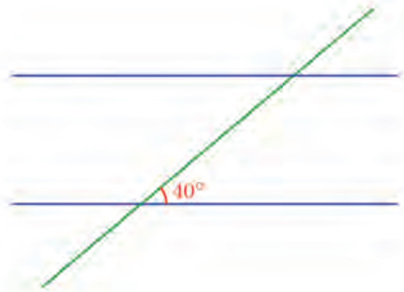


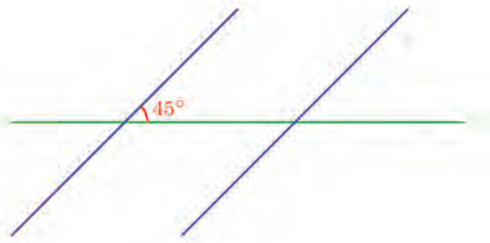
The blue lines in the figure appear to be parallel. As there is a difference of 1° in the slants, they will meet when extended sufficiently. We can calculate how much to be extended. You've to extend them by more than a metre for them to meet!

Draw two parallel lines and an intersecting line as in the earlier activity. Mark the eight angles around the points of intersection. You may hide their measures (Right click and uncheck **Show Label** box). Give the same colour to the four small angles of the same measure **(Right Click → Object Properties → Colour)** Choose the colour you want. You can change **Opacity**. In the same way, give another colour to all the large angles of the same measure.



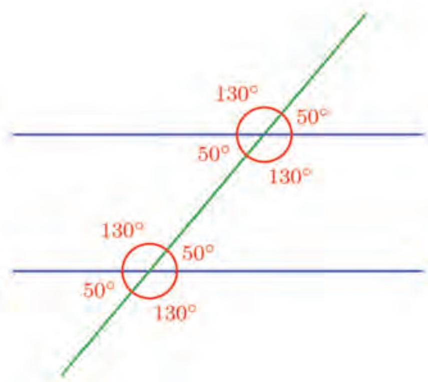
In each of the pictures below, can you calculate the other seven angles which the parallel blue lines make with the green line ?





Matching angles

We have seen the relations between the four angles made by two intersecting lines. What can we say about the relation between the eight angles formed when a line cuts two parallel lines ?



Let's take another look at this figure which we saw earlier.

We know the relation between the four angles below. Same is the relation between the four angles above.

What if we take an angle from below and an angle from above ?

If both are small angles, each is 50°.

If both are large, each is 130°.

If one is small and the other is large; the small one is 50°, the large one 130°; and the sum is 180°.

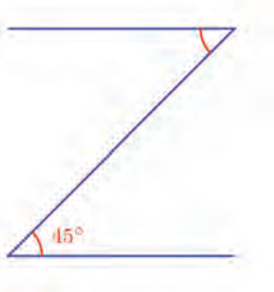
The relations remain the same even if the angles change, right ? So we can say this in general :

Of the angles made when two parallel lines are cut by a slanting line,

- the small angles are of the same measure.
- the large angles are of the same measure.
- a small angle and a large angle add up to 180°.

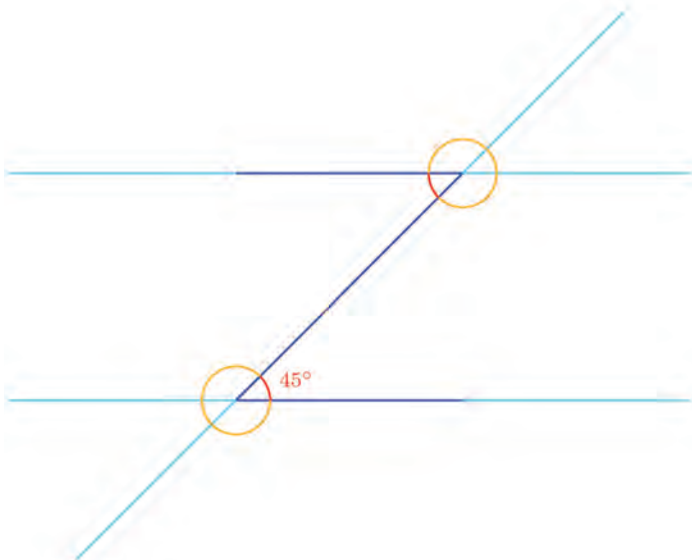
If the intersecting line is perpendicular to one of the parallel lines, it would be perpendicular to the other line too, and all angles would be right angles.

Now, look at this figure.



The top and bottom lines are parallel. What is the measure of angle above ?

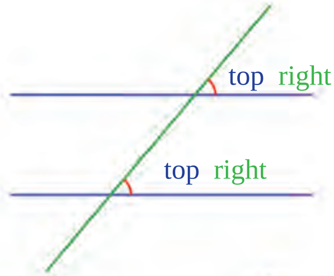
To see all the angles clearly, let's suppose the lines are extended.

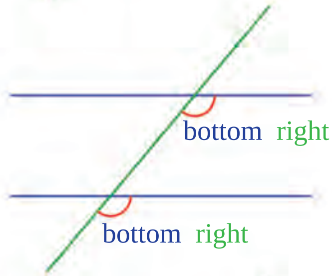


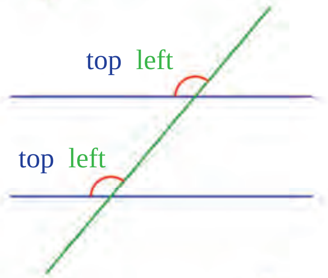
Look at the two small angles formed when the slanting line cuts the top and bottom parallel lines. These are the angles in the first figure. So they are of the same measure.

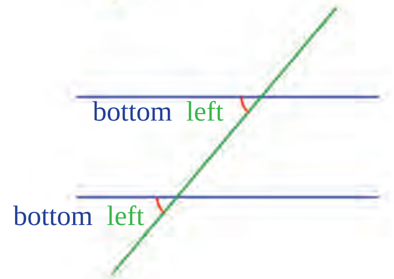
Position and angle

We may group the angles formed when two parallel lines are cut by another line based on their position. The figures below show angle pairs in the same position.





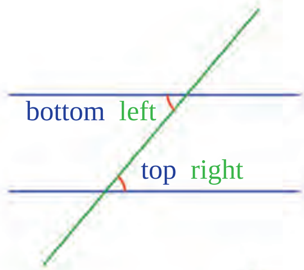


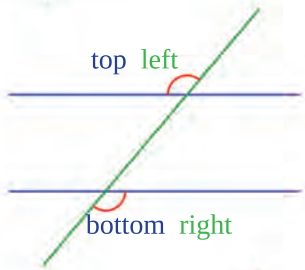


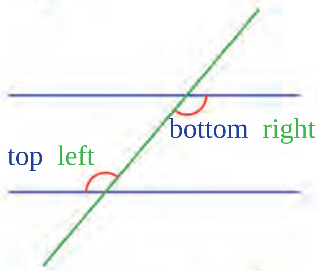
Angles in each such pair are called corresponding angles. The angles in each pair measure the same.

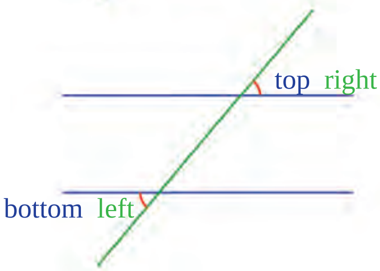
Opposite positions

The figures below show the angles in opposite positions when two parallel lines are cut by another line



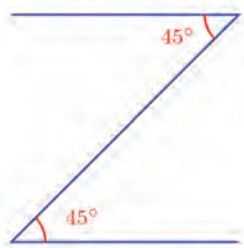




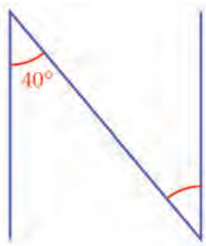


Angles in each such pair are called alternate angles. The angles in each pair measure the same.

That is, the angle above is also 45°.



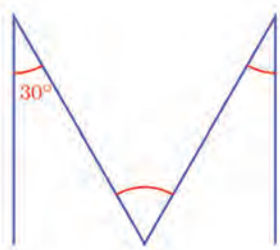
What if the figure looks like this ?



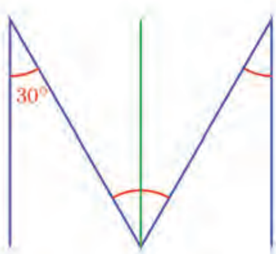
This is the first figure with the angle slightly changed and turned a bit, isn't it ?

What is the measure of the other angle ?

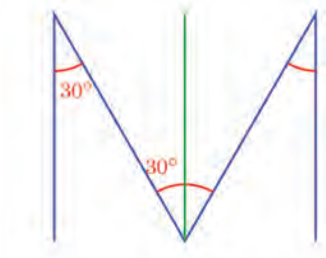
Now look at this figure :



The vertical lines are parallel. But there is no line cutting them. How about drawing another vertical line ?

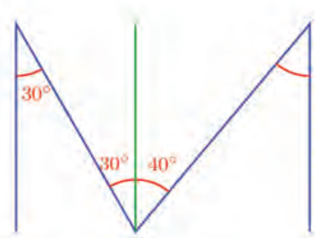


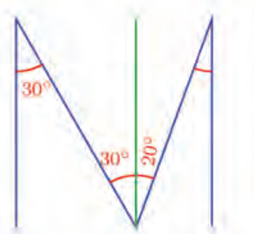
Now the, middle angle is in two parts. We can find the left part.



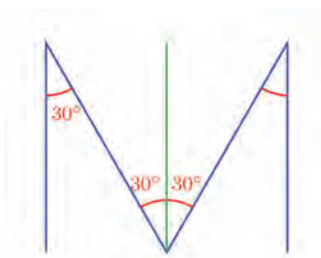
What about the right part ?

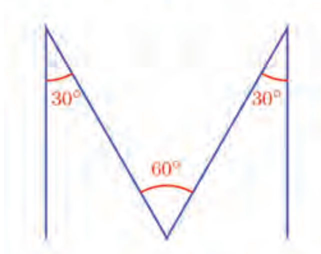
Let's try different measures for this angle:





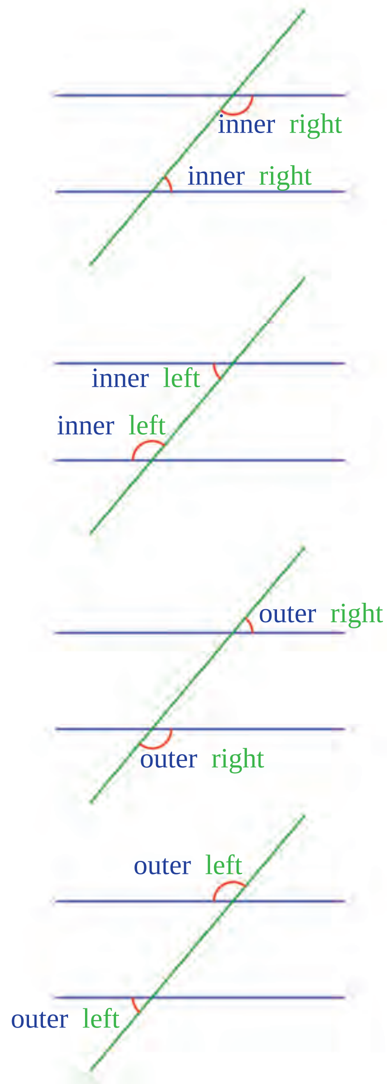
What should be this angle to get a nice figure?





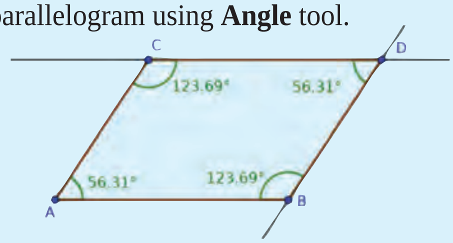
In and out

The figures below show the interior and exterior angle pairs when two parallel lines are cut by another line.



The first two pairs are called co-interior angles and the last two pairs co-exterior angles. The sum of the angles in each such pair is 180°.

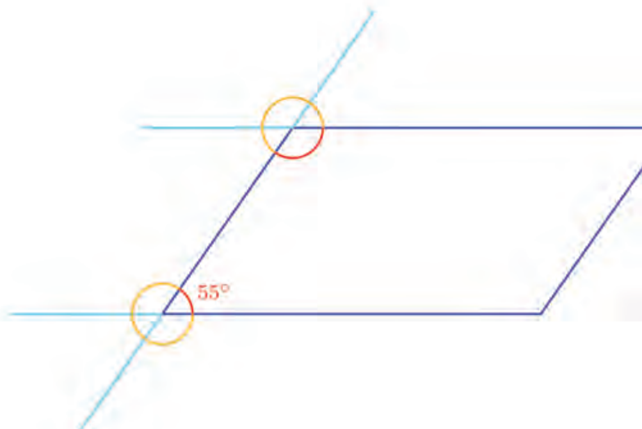
Let's draw a parallelogram. Draw two lines AB and AC. Through B draw a line parallel to AC and through C draw a line parallel to AB. The point of intersection of these lines is D. Draw parallelogram $ABDC$ using **Polygon** tool. We can see all angles if we click in the parallelogram using **Angle** tool.



What is the relation between these angles? Try changing the position of C. Do the angles change ? And the relation between them ?

Another question : Can you find out the other angles in the given parallelogram ?

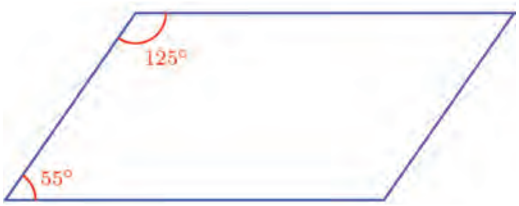
First, take the angle above the 55° angle. To determine this, we shall look at the angles of intersection of the left side with the top and bottom parallel lines.



The 55° angle and the angle above it form a pair of small angle and large angle.

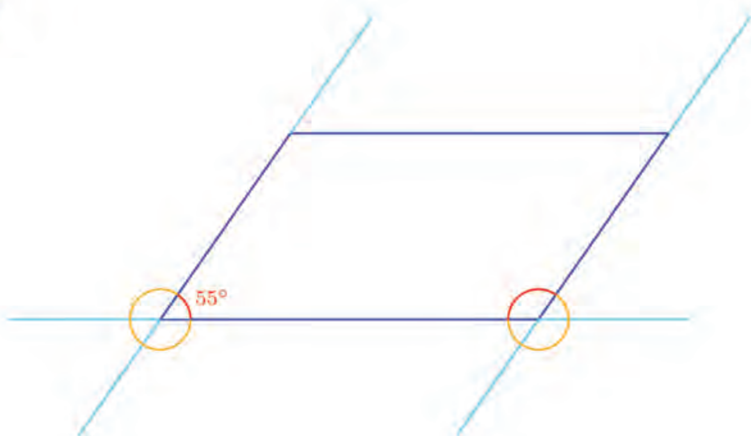
So their sum is 180°.

This means the top angle = 180° – 55° = 125°.



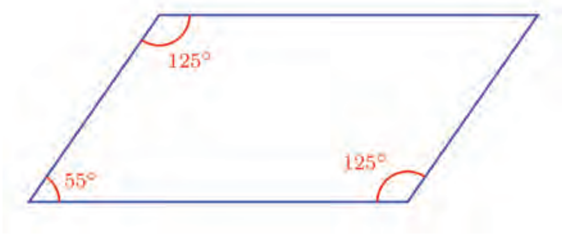
Now look at the angle to the right of the marked angle.

To calculate this, look at the angles made by the left and right parallel sides with the bottom line.



The 55° angle and the angle on its right are a small angle and a large angle of these angles.

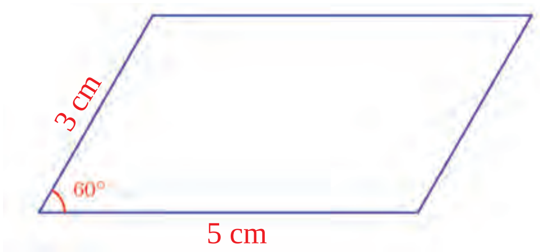
So, this angle also is 125° as calculated earlier.



Can't you find the fourth angle, like this ?

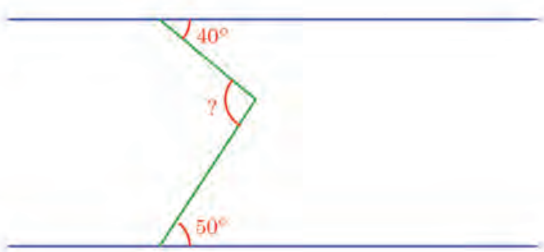
Now try these problems:

(1) Draw the parallelogram below with the given measures.

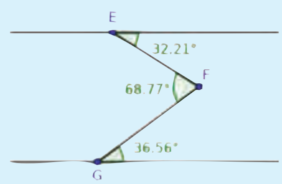


Calculate the other three angles.

(2) The top and bottom blue lines in the figure are parallel. Find the angle between the green lines.



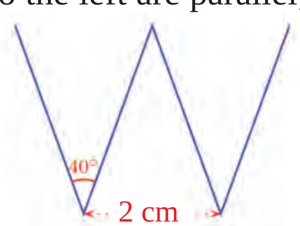
Draw two parallel lines. Mark a point on each. Mark a third point in between them. Draw lines joining the points on the lines with the third point. Mark the angles which these lines make with the parallel lines. Also mark the angle between the lines.



What is the relation between the three angles? Try changing the position of the points. Does the relation between the angles remain the same ?

(3) In the figure, the pair of lines slanted to the left are parallel; and also the pair of lines slanted to the right.

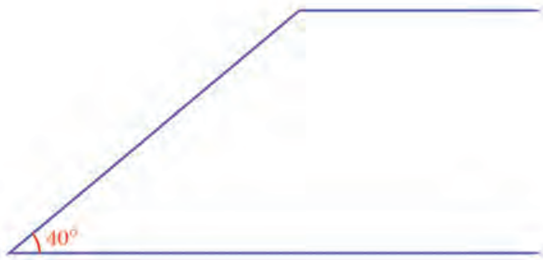
Draw this figure :



ST-359-2-MATHS (E)-7-VOL-1

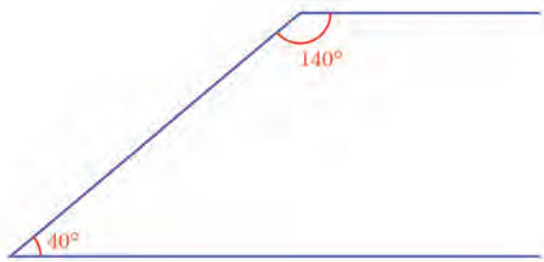
Triangle sum

Look at this figure :



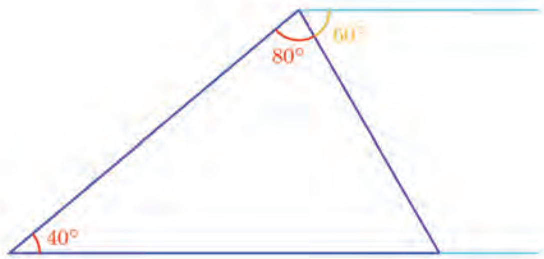
The top and bottom lines are parallel.

So, can you calculate the angle at the top ?



If this angle is drawn less than 140°, then the two lines will meet.

Let's decrease it by 60°.



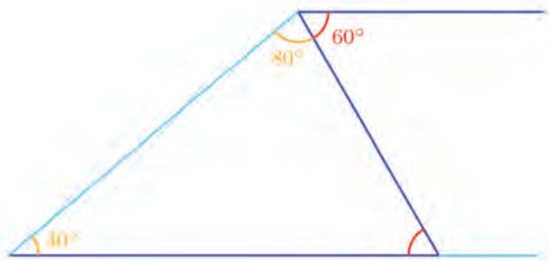
Now we have a triangle.

What are the angles in this triangle ?

The left angle is 40°. The top angle is 140° – 60° = 80°.

What about the third angle ?

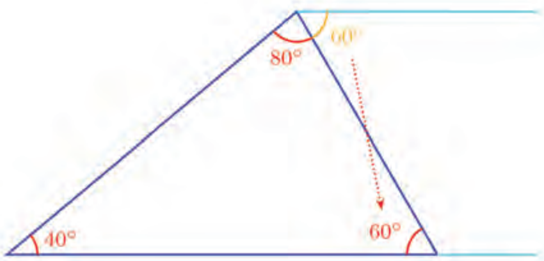
It is one of the small angles which the new slanted line makes with the parallel lines .



Its measure is the same as that of the small angle which this line makes with the top line.

Isn't the top small angle 60° ?

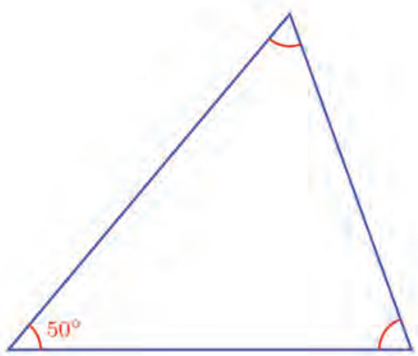
So the bottom small angle is also 60°.



Thus the 60° which we took away from the top reappears at the bottom as an angle of the triangle.

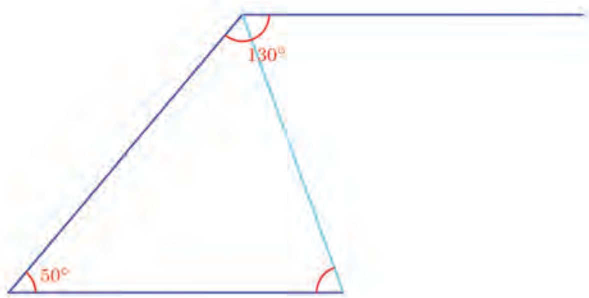
The sum of this angle and the top angle of the triangle 80° + 60° = 140°.

Now look at this figure:

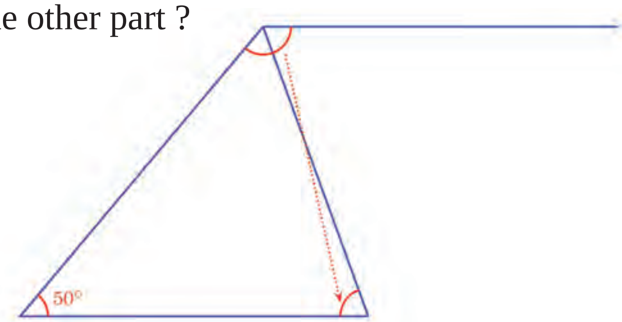


Can you calculate the sum of the other two angles of this triangle ?

In the first problem, the right side of the triangle was got by drawing a slanted line instead of the parallel line. Let's think in the reverse. A parallel line instead of the slanted right side. What is the angle which this parallel line makes with the left side ?



When we drew the triangle, this angle split into two. One part is the top angle of the triangle. What about the other part ?



That is, one part of the 130° angle is the top angle of the triangle and the other part is the angle on the right in the triangle.

So, the sum of these two angles of the triangle is 130°.

How do we state in general what we've learnt from this problem ?

If we subtract the measure of one angle of a triangle from 180°, we get the sum of the other two angles.

Using the **Polygon** tool, draw a triangle and mark the angles (click inside the triangle using the **Angle** tool). What is the sum of all angles ? Change the vertices of the triangle and check.

For example, if one angle of a triangle is 60°, the sum of the other two angles:

180° – 60° = 120°

What about the sum of the three angles of the triangle ?

This is true for any triangle.

The sum of all angles of a triangle is 180°

Now try this problem :

One angle of a right triangle is 40°. What is the measure of the angle other than the right angle ?

Let's think this way. The sum of the angles other than the right angle is

$$180° - 90° = 90°$$

One of them is 40°. Then the other angle is

$$90° - 40° = 50°$$

We can think in another way as well. The sum of the three angles is 180°. The sum of two of them

$$90° + 40° = 130°$$

So the third angle

$$180° - 130° = 50°$$

Another problem :

One angle of a triangle is 72°. The other two angles are of equal measure. What are their measures ?

What is the sum of the other two angles ?

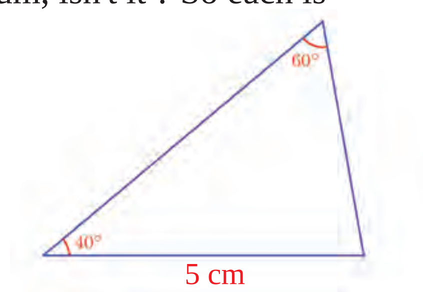
$$180° - 72° = 108°$$

Since the other two angles are equal, each is half the sum, isn't it ? So each is

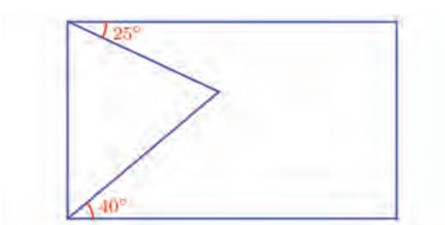
$$\frac{108°}{2} = 54°$$

Now try these problems :

(1) Draw the triangle with the given measures.

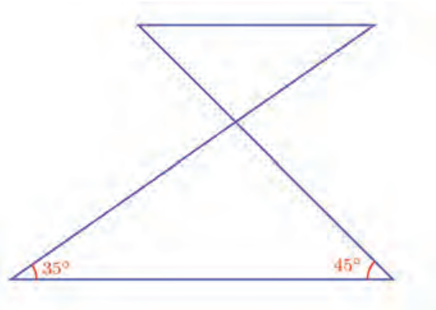


(2) The figure shows a triangle drawn in a rectangle.



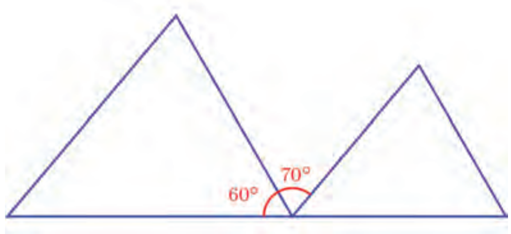
Calculate the angles of the triangle.

(3) The top and bottom lines in the figure are parallel.



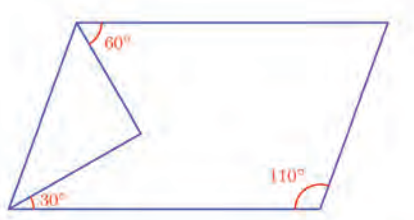
Calculate the third angle of the bottom triangle and all angles of the top triangle.

(4) The left and right sides of the large triangle are parallel to the left and right sides of the small triangle.



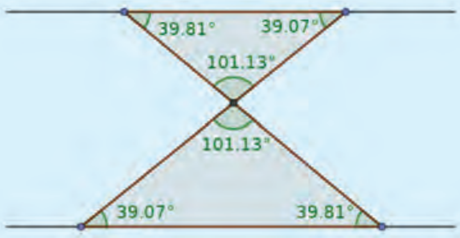
Calculate the other two angles of the large triangle and all angles of the small triangle.

5. A triangle is drawn inside a parallelogram.



Calculate the angles of the triangle.

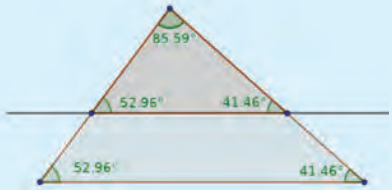
Draw two parallel lines and mark two points on each. Join them as in the figure and mark the point of intersection. Use the **Polygon** tool to draw two triangles and mark their angles.



What is the relation between the angles of the triangle ?

Try changing the positions of the points.

Draw a triangle and mark a point on one of the sides. Draw a line through this point parallel to another side of the triangle. Mark the point where this line meets the third side. Draw the small triangle with one corner of the triangle and the points on the sides as vertices. Mark the angles of the first triangle and the small triangle. What is the relation between these angles ? Try changing the corners of the triangle.



2 FRACTIONS

Multifold multiplication

A packet of sugar weighs two kilograms. How many kilograms is four such packets ?

Let's do some mental math. Four times two kilograms is eight kilograms.

In detail,

$$4 \times 2 = 2 + 2 + 2 + 2 = 8$$

What if they are half kilogram packets ?

One kilogram in two packets; so two kilograms in four packets.

That is, four times half a kilogram is two kilograms.

In terms of numbers alone, four times half is two.

Just as we wrote four times two as 4×2, we can write four times half as $4 \times \frac{1}{2}$.

That is,

$$4 \times \frac{1}{2} = 4 \text{ times } \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2$$

A bottle holds a quarter litre of water. How much water is needed to fill three such bottles ?

Three times a quarter litre is three quarters of a litre.

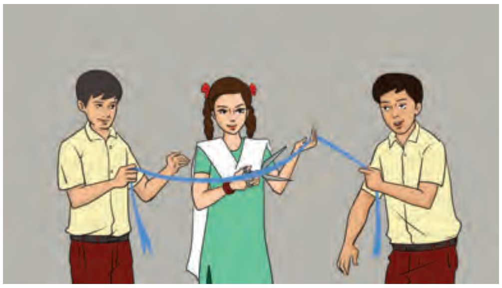
In terms of numbers alone,

$$3 \text{ times } \frac{1}{4} \text{ is } \frac{3}{4}$$

As a product,

$$3 \times \frac{1}{4} = 3 \text{ times } \frac{1}{4} = \frac{3}{4}$$

Another problem : Five strings, each a quarter metre long are placed end to end. What is the total length ?



Four quarters make one; and another quarter makes it one and a quarter. Total length is one and a quarter metres. That is,

$$5 \text{ times } \frac{1}{4} \text{ is } 1\frac{1}{4}$$

As a product ?

$$5 \times \frac{1}{4} = 1\frac{1}{4}$$

Do the following problems mentally. Write each as how many times and also as a product.

(1) Each piece of a pumpkin weighs a quarter kilogram. What is the weight of two pieces together ? What is the weight of four such pieces ? Six pieces ?

(2) We can fill a cup with one third of a litre of milk. How much milk is needed to fill two cups ? Four cups ?

(3) What is the total length of four pieces of ribbons, each of length three fourths of a metre? What about five pieces ?

(4) It takes $\frac{1}{4}$ hour to walk around a play ground once.

(i) How much time does it take to walk 4 times around at this speed ?

(ii) What about 7 times ?

Let's look at the calculations in such problems:

What is 2 times $\frac{1}{3}$?

$$2 \times \frac{1}{3} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

What about 3 times ?

$$3 \times \frac{1}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

To get 4 times of $\frac{1}{3}$, we need just add a $\frac{1}{3}$ to it, isn't it ?

$$4 \times \frac{1}{3} = \left(3 \times \frac{1}{3}\right) + \frac{1}{3} = 1\frac{1}{3}$$

It can be done this way also:

$$4 \times \frac{1}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{1+1+1+1}{3} = \frac{4}{3} = 1\frac{1}{3}$$

How do we calculate 4 times $\frac{2}{3}$?

$$4 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{2+2+2+2}{3} = \frac{4 \times 2}{3} = \frac{8}{3}$$

Splitting 8 as a multiple of 3 and remainder,

$$\frac{8}{3} = \frac{6+2}{3} = \frac{6}{3} + \frac{2}{3} = 2\frac{2}{3}$$

What about 10 times $\frac{2}{3}$?

We need to add ten $\frac{2}{3}$'s

$$10 \times \frac{2}{3} = \frac{10 \times 2}{3} = \frac{20}{3} = \frac{18+2}{3} = \frac{18}{3} + \frac{2}{3} = 6\frac{2}{3}$$

Now try this problem :

$\frac{3}{4}$ litres of milk in a bottle; how many litres in 7 such bottles ?

We need to find 7 times $\frac{3}{4}$

$$7 \times \frac{3}{4} = \frac{7 \times 3}{4} = \frac{21}{4}$$

Splitting 21 as a multiple of 4 and remainder,

$$\frac{21}{4} = \frac{20+1}{4} = \frac{20}{4} + \frac{1}{4} = 5\frac{1}{4}$$

So, $5\frac{1}{4}$ litres in 7 bottles.

Share and fraction

If 4 litres of milk is divided equally among 3 persons, how much of milk will each one get ?

First give 1 litre to each one. If the remaining 1 litre is divided among three persons, each will get $\frac{1}{3}$ litre more. In total $1\frac{1}{3}$ litres.

Here since 4 is divided into 3, we can write it as a division.

$$4 \div 3 = 1\frac{1}{3}$$

Also as a fraction.

$$\frac{4}{3} = 1\frac{1}{3}$$

Now try these problems.

(1) The weight of an iron block is $\frac{1}{4}$ kilogram

(i) What is the total weight of such 15 blocks ?

(ii) 16 blocks ?

(2) Some 2 metre long rods are cut into 5 pieces of equal length.

(i) What is the length of each piece ?

(ii) What is the total length of 4 pieces ?

(iii) Of 10 pieces?

(3) 5 litres of milk is filled in 6 bottles of the same size.

(i) How many litres of milk does each bottle hold ?

(ii) How many litres in 3 bottles together ?

(iii) In 4 bottles ?

Part and multiplication

A six metre long ribbon is cut into two equal pieces. What is the length of each piece?

Half of six metres, that is three metres.

Half means one of two equal parts, that is $\frac{1}{2}$.

So $\frac{1}{2}$ of 6 metres is 3 metres.

In terms of numbers alone,

$$\frac{1}{2} \text{ of } 6 \text{ is } 3$$

Just as in the case of multiples, parts can also be written as products. That is,

$$\frac{1}{2} \times 6 = \frac{1}{2} \text{ of } 6 = 3$$

Suppose we divide a six metre long ribbon into three equal pieces ?

The length of each piece is a third of six metres; that is two metres

$$\frac{1}{3} \text{ of } 6 \text{ metres is } 2 \text{ metres}$$

In terms of numbers alone,

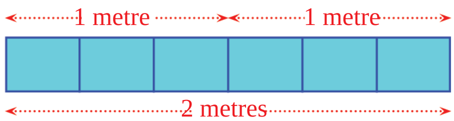
$$\frac{1}{3} \text{ of } 6 \text{ is } 2$$

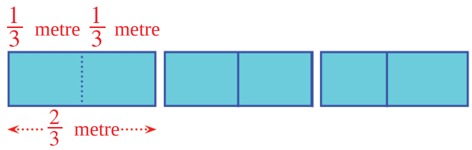
Writing it as a product,

$$\frac{1}{3} \times 6 = 2$$

What if we divide a two metre long string into three equal pieces ?

We can divide each metre into three equal parts and then take two parts together.





Then,

$$\frac{1}{3} \text{ of 2 metres is } \frac{2}{3} \text{ metre}$$

In terms of numbers alone,

$$\frac{1}{3} \text{ of } 2 \text{ is } \frac{2}{3}$$

Writing it as a product,

$$\frac{1}{3} \text{ of } 2 = \frac{1}{3} \times 2 = \frac{2}{3}$$

Another problem :

What is a quarter of five kilogram ?

A quarter of four kilograms is one kilogram; and then a quarter of the remaining one kilogram. So one and a quarter kilogram in all. In terms of numbers,

$$\frac{1}{4} \text{ of } 5 \text{ is } 1\frac{1}{4}$$

$$\frac{1}{4} \text{ of } 5 = \frac{1}{4} \times 5$$

$$= 1\frac{1}{4}$$

Parts and times

If three litres of milk is divided between four people, how much will each get ?

One fourth of three litres; that is three quarters of a litre. There's another way of thinking. When one litre of milk is divided among four people, each gets a quarter of a litre. Because it is three litres, this can be done three times. So one gets three times a quarter of a litre which is three quarters of a litre.

That is, one fourth of three litres and three times a quarter of a litre are both three quarters of a litre.

As products of numbers,

$$\frac{1}{4} \times 3 = 3 \times \frac{1}{4}$$

Do these problems in head. Then write each as a part and also as a product of numbers.

(1) Nine litres of milk is divided equally among three children. How many litres will each get ? What if there are four children ?

(2) Six kilograms of rice was packed in five bags of the same size. How many kilograms of rice in each bag ? What if it is packed in four bags ?

(3) A seven metre long string is divided into six equal pieces. What is the length of each piece ? What if it is divided into three equal pieces ?

Let's look at the calculations to find parts of numbers.

What is $\frac{1}{4}$ of 8 ?

When 8 is divided into 4 equal parts, each part is 2.

Writing as a product,

$$\frac{1}{4} \times 8 = \frac{8}{4} = 2$$

What about $\frac{3}{4}$ of 8 ?

8 divided into 4 equal parts, and 3 of these parts taken together.

In other words, 3 times what is got by dividing 8 into 4 equal parts.

That is, 3 times 2, which is 6.

Writing as a product,

$$\frac{3}{4} \times 8 = 3 \times \frac{8}{4} = 3 \times 2 = 6$$

It can be done this way also.

$$\frac{3}{4} \times 8 = 3 \times \frac{8}{4} = \frac{24}{4} = 6$$

What is $\frac{3}{4}$ of 9 ?

$$\frac{3}{4} \times 9 = 3 \times \frac{9}{4}$$

Here, instead of writing $\frac{9}{4} = 2\frac{1}{4}$ and proceeding, it is easier to do like this:

$$\frac{3}{4} \times 9 = 3 \times \frac{9}{4} = \frac{3 \times 9}{4} = \frac{27}{4}$$

To see what this actually means, we write 27 as a multiple of 4, and remainder :

$$\frac{27}{4} = \frac{24+3}{4} = \frac{24}{4} + \frac{3}{4} = 6\frac{3}{4}$$

Now see this problem:

We have to cut off $\frac{3}{5}$ of a 7 metre long string. How long is this piece ?

We need to calculate $\frac{3}{5}$ of 7. How we do this ?

$$\frac{3}{5} \times 7 = \frac{3\times 7}{5} = \frac{21}{5} = \frac{20+1}{5} = \frac{20}{5} + \frac{1}{5} = 4\frac{1}{5}$$

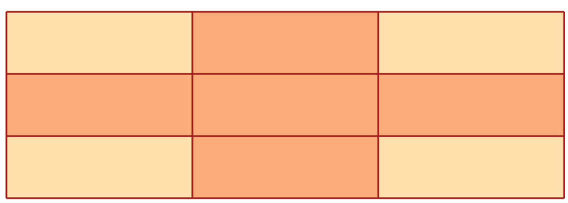
We need to cut off $4\frac{1}{5}$ metres (It is easy to cut it if we say this as 4 metres, 20 centimetres).

Now try these problems:

(1) There are 35 children in a class. $\frac{3}{5}$ of them are girls. How many girls are there in the class ?

(2) 10 kilograms of rice is filled equally in 8 bags. If the rice in 3 such bags are taken together, how many kilograms would that be ?

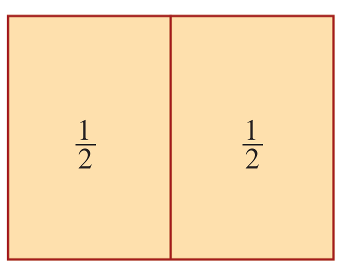
(3) The area of the rectangle in the figure is 27 square centimetres. It is divided into 9 equal parts.



What is the area of the darker part in square centimetres ?

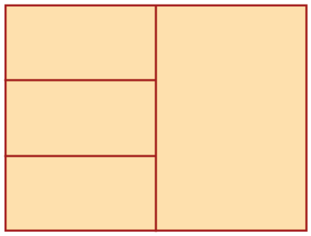
Part of part

Look at this figure:

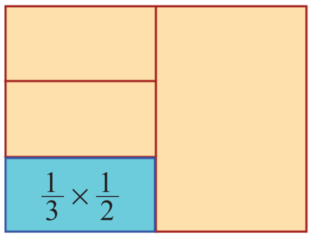


A rectangle is divided into two equal parts. Each part is half of the large rectangle, that is, $\frac{1}{2}$ of the large rectangle.

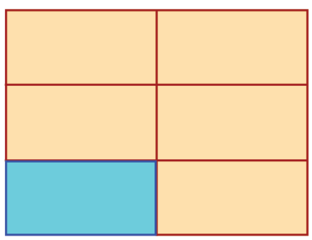
Now, suppose we divide one of these parts into three equal parts.



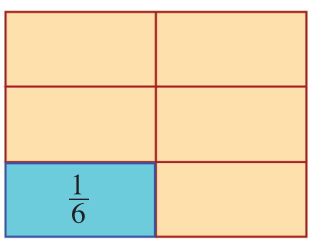
Each of these three small rectangles is $\frac{1}{3}$ of $\frac{1}{2}$ of the large rectangle, isn't it ? That is, $\frac{1}{3} \times \frac{1}{2}$ of it.



To look at it another way, suppose we extend the horizontal lines :



What part of the large rectangle is each of these 6 small rectangles ?

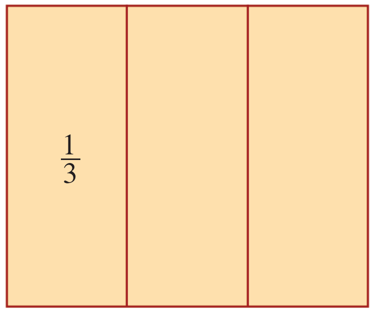


Now, what do we get ?

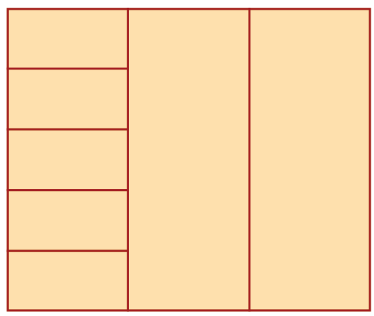
$$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

Can we calculate $\frac{1}{5}$ of $\frac{1}{3}$ like this ? That is $\frac{1}{5} \times \frac{1}{3}$?

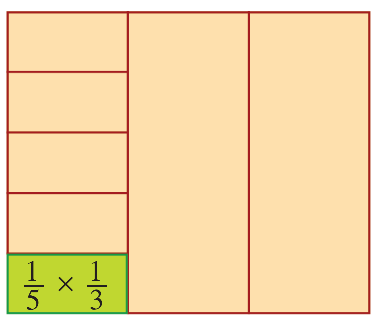
Let's draw a rectangle and divide it into three equal parts :



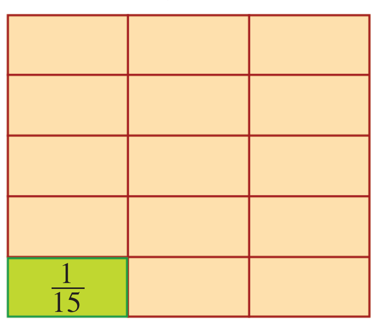
Now we draw horizontal lines to divide one of these rectangles into 5 equal parts.



Each of these small rectangles is $\frac{1}{5}$ of $\frac{1}{3}$ of the largest rectangle; that is, $\frac{1}{5} \times \frac{1}{3}$ of it.



Suppose we extend the horizontal lines.



So, $\frac{1}{5} \times \frac{1}{3} = \frac{1}{15}$

Now try these problems:

(1) Draw rectangles and find these products.

i) $\frac{1}{2} \times \frac{1}{4}$

(ii) $\frac{1}{3} \times \frac{1}{6}$

(iii) $\frac{1}{5} \times \frac{1}{8}$

(2) A one metre long string is divided into five equal parts. How long is half of each part in metres ? In centimetres?

(3) One litre of milk is filled in two bottles of equal size. A quarter of the milk in one bottle was used to make tea. How many litres of milk were used for tea ? In millilitres ?

Another type of problem: what is $\frac{1}{5}$ of $\frac{2}{3}$?

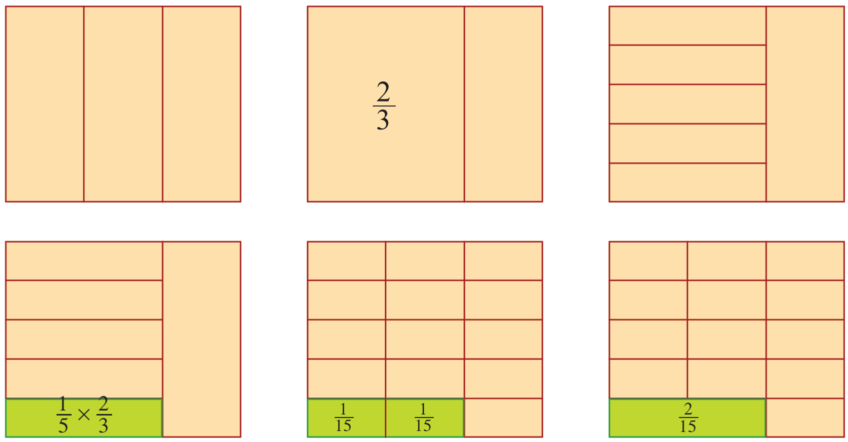
Let's think this way :

- $\frac{2}{3}$ means $\frac{1}{3}$ of 2
- $\frac{1}{5}$ of $\frac{2}{3}$ is $\frac{1}{5}$ of $\frac{1}{3}$ of 2
  - $\frac{1}{5}$ of $\frac{1}{3}$ is $\frac{1}{15}$
  - $\frac{1}{5}$ of $\frac{1}{3}$ of 2 is $\frac{1}{15}$ of 2
- $\frac{1}{5}$ of $\frac{2}{3}$ is $\frac{2}{15}$

As a product,

$$\frac{1}{5} \times \frac{2}{3} = \frac{1}{5} \times \frac{1}{3} \times 2 = \frac{1}{15} \times 2 = \frac{2}{15}$$

We can draw a rectangle to find $\frac{1}{5}$ of $\frac{2}{3}$ directly :



What about $\frac{4}{5}$ of $\frac{2}{3}$, that is $\frac{4}{5} \times \frac{2}{3}$?

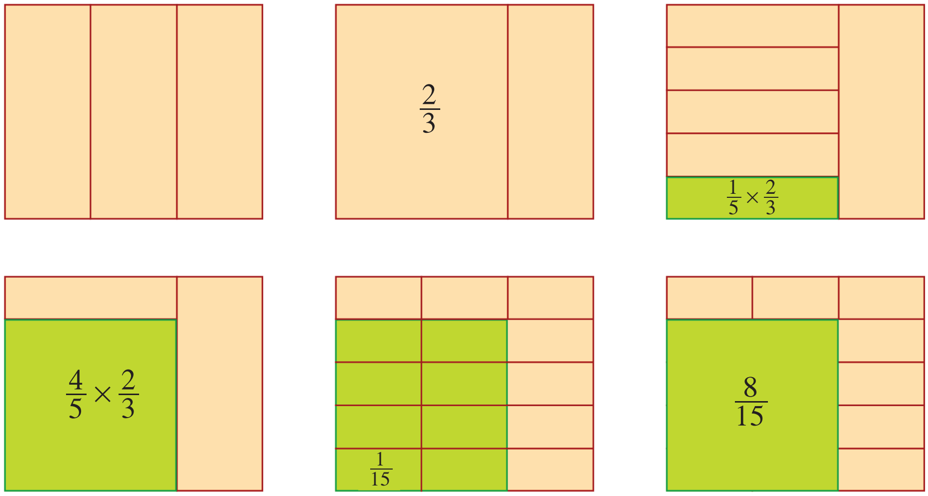
We first find $\frac{1}{5}$ of $\frac{2}{3}$ and then find 4 times that.

$$\begin{aligned}
\frac{4}{5} \times \frac{2}{3} &= 4 \times \frac{1}{5} \times \frac{2}{3} \\
&= 4 \times \frac{1}{5} \times \frac{1}{3} \times 2 \\
&= 4 \times \frac{2}{15} \\
&= \frac{8}{15}
\end{aligned}$$

We can do it like this also:

$$\begin{aligned}
\frac{4}{5} \times \frac{2}{3} &= 4 \times \frac{1}{5} \times \frac{1}{3} \times 2 \\
&= 4 \times \frac{1}{5 \times 3} \times 2 \\
&= 4 \times \frac{1}{15} \times 2 \\
&= 4 \times \frac{2}{15} \\
&= \frac{8}{15}
\end{aligned}$$

Again we can draw a picture to show this :



Now try these problems :

(1) A rope 2 metres long is cut into 5 equal pieces. What is the length of three quarters of one of the pieces in metres ? In centimetres ?

(2) 4 bottles of the same size were filled with 3 litres of water. One of these was used to fill 5 cups of the same size. How much water is there in one such cup, in litres ? And in millilitres ?

(3) A watermelon weighing four kilograms was cut into five equal pieces. One piece was again halved. What is the weight of each of these two pieces in kilograms ? And in grams ?

(4) A vessel full of milk is used to fill three bottles of the same size. Then the milk in each bottle was used to fill four cups of the same size. What fraction of the milk in the first vessel does each cup contain ?

(5) Draw a line *AB* of length 12 centimetres. Mark *AC* as $\frac{2}{3}$ of *AB*. Mark *AD* as $\frac{1}{4}$ of *AC*. What part of *AB* is *AD* ?

(6) Calculate the following using multiplication :

(i) $\frac{3}{7}$ of $\frac{2}{5}$ (ii) $\frac{3}{5}$ of $\frac{2}{7}$

(iii) $\frac{2}{3}$ of $\frac{3}{4}$ (iv) $\frac{5}{6}$ of $\frac{3}{10}$

More on multiplication

A bottle can contain one and a half litres of water. Four such bottles of water was poured into a vessel. How much water is there in the vessel ?

When we pour two bottles it is three litres; four bottles makes it six litres, isn't it ? Here we've found 4 times $1\frac{1}{2}$. Writing this as a product,

$$4 \times 1\frac{1}{2} = 6$$

Suppose we poured the water in 3 bottles, each containing $2\frac{1}{4}$ litres, into the vessel ?

If it is 2 litre bottles, then 6 litres. Here we have $\frac{1}{4}$ litre more in each bottle. So we must add $\frac{3}{4}$ litre also. That is $6\frac{3}{4}$ litres.

Writing this as a product,

$$\begin{aligned} 3 \times 2\frac{1}{4} &= 3 \times \left(2 + \frac{1}{4}\right) \\ &= (3 \times 2) + \left(3 \times \frac{1}{4}\right) \\ &= 6 + \frac{3}{4} \\ &= 6\frac{3}{4} \end{aligned}$$

There is another way to calculate this. We can write $2\frac{1}{4}$ litres as $\frac{9}{4}$ litres. Then,

$$\begin{aligned} 3 \times 2\frac{1}{4} &= 3 \times \frac{9}{4} \\ &= 3 \times \frac{1}{4} \times 9 \\ &= \frac{3}{4} \times 9 \\ &= \frac{27}{4} \\ &= 6\frac{3}{4} \end{aligned}$$

Like this, in what all ways can you calculate 5 times $3\frac{1}{2}$?

We can do it this way :

$$\begin{aligned} 5 \times 3\tfrac{1}{2} &= 5 \times \left(3 + \tfrac{1}{2}\right) \\ &= (5 \times 3) + \left(5 \times \tfrac{1}{2}\right) \\ &= 15 + 2\tfrac{1}{2} \\ &= 17\tfrac{1}{2} \end{aligned}$$

And this is another way :

$$\begin{aligned} 5 \times 3\tfrac{1}{2} &= 5 \times \tfrac{7}{2} \\ &= \frac{5 \times 7}{2} \\ &= \frac{35}{2} \\ &= 17\tfrac{1}{2} \end{aligned}$$

Let's look at another thing : Six metres is thrice two metres. What about seven metres ? It's thrice two metres and a metre more. In other words, thrice two metres and half of two metres.

So we may say that seven metres is three and a half of two metres.

As a product,

$$3\tfrac{1}{2} \times 2 = 7$$

How did we get this ?

$$\begin{aligned} 3\tfrac{1}{2} \times 2 &= \left(3 + \tfrac{1}{2}\right) \times 2 \\ &= (3 \times 2) + \left(\tfrac{1}{2} \times 2\right) \\ &= 6 + 1 \\ &= 7 \end{aligned}$$

In the same way, two and a quarter times five means, twice five together with a quarter of five. That is, ten and one and a quarter; makes eleven and a quarter.

As a product,

$$
\begin{aligned}
2\tfrac{1}{4} \times 5 &= \left(2 + \tfrac{1}{4}\right) \times 5 \\
&= (2 \times 5) + \left(\tfrac{1}{4} \times 5\right) \\
&= 10 + 1\tfrac{1}{4} \\
&= 11\tfrac{1}{4}
\end{aligned}
$$

Another way to do this is :

$$
\begin{aligned}
2\tfrac{1}{4} \times 5 &= \frac{9}{4} \times 5 \\
&= \frac{9 \times 5}{4} \\
&= \frac{45}{4} \\
&= 11\tfrac{1}{4}
\end{aligned}
$$

How do we calculate $3\frac{1}{2}$ times $2\frac{1}{4}$? Writing $2\frac{1}{4}$ as $\frac{9}{4}$ and $3\frac{1}{2}$ as $\frac{7}{2}$,

$$3\tfrac{1}{2} \times 2\tfrac{1}{4} = \frac{7}{2} \times \frac{9}{4} = \frac{7 \times 9}{2 \times 4} = \frac{63}{8} = \frac{56+7}{8} = 7\tfrac{7}{8}$$

Now try these problems yourself :

(1) One and a half metres of cloth is needed for a shirt. How much cloth is required for five such shirts ?

(2) The price of one kilogram of okra is thirty rupees. What is the price of two and a half kilograms ?

(3) A person walks two and a half kilometres in an hour. At the same speed, how far will he walk in one and a half hours ?

(4) Roni has 36 stamps with her. Sahira says she has $2\frac{1}{2}$ times this. How many stamps does Sahira have ?

(5) Joji works $4\frac{1}{2}$ hours each day. How many hours does he work in 6 days ?

(6) Calculate the following :

(i) 4 times $5\frac{1}{3}$

(ii) $4\frac{1}{3}$ times 5

(iii) $1\frac{1}{2}$ times $\frac{2}{3}$

(iv) $\frac{2}{5}$ times $2\frac{1}{2}$

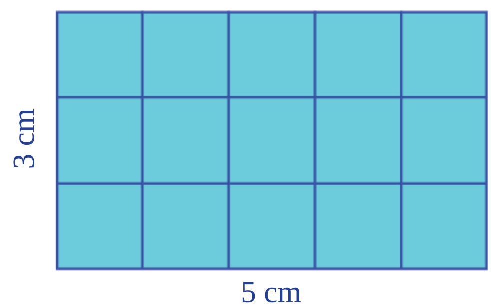
(v) $2\frac{1}{2}$ times $5\frac{1}{2}$

Fractional area

We've studied areas of rectangles in class 5.

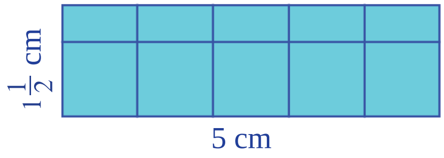
A rectangle is 5 centimetres long and 3 centimetres high. What is its area in square centimetres ?

We calculated this area by filling this rectangle with squares of side one centimetre.

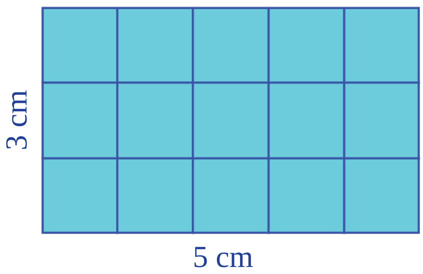


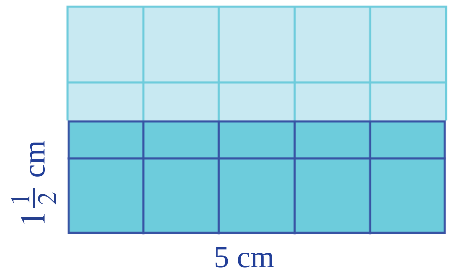
$5 \times 3 = 15$ such squares fill the rectangle. So, area is 15 square centimetres.

What happens if the sides are of length 5 centimetres and $1\frac{1}{2}$ centimetres ?



Isn't this half the first rectangle ?





So the area is half of 15, that is $7\frac{1}{2}$ square centimetres.

This may be done in another way. What is the area of each small rectangle in the top row of the picture on the right ?

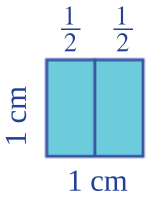
Each one is half the area of the square with area 1 square centimetre, isn't it ?

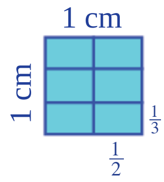
So we can say that the area of each rectangle is $\frac{1}{2}$ square centimetre.

Taking five of them together, the total area is $5 \times \frac{1}{2} = 2\frac{1}{2}$ square centimetres. So the area of the large rectangle is $5 + 2\frac{1}{2} = 7\frac{1}{2}$ square centimetres.

How do we calculate the area of a rectangle with sides of length $\frac{1}{2}$ centimetre and $\frac{1}{3}$ centimetre ?

To draw such a rectangle, we first draw a square of side 1 centimetre. Then we draw one vertical line and two horizontal lines to split it into 6 equal parts.

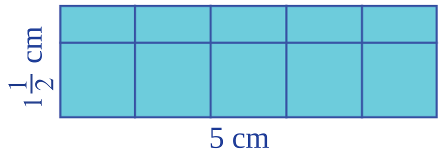




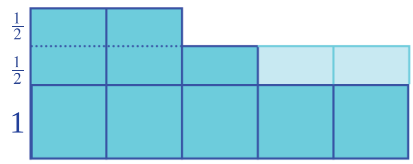
Each small rectangle has sides $\frac{1}{2}$ centimetre and $\frac{1}{3}$ centimetre. What about its area ?

One and half

We halved a rectangle of length 5 centimetres and height 3 centimetres. This is the result.



Suppose we rearrange the half squares in the top row this way ?



Area = $5 + 2\frac{1}{2} = 7\frac{1}{2}$ sq.cm.

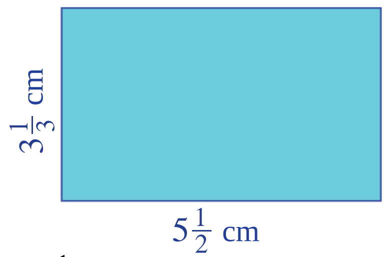
Each of them is $\frac{1}{6}$ of the large square. The area of the square is 1 square centimetre. So the area of the small rectangle is $\frac{1}{6}$ square centimetre.

What about the area of a rectangle with sides of length $\frac{1}{3}$ centimetre and $\frac{1}{5}$ centimetre ?

Dividing the square of side 1 centimetre in another way, we find this area to be

$$\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$$

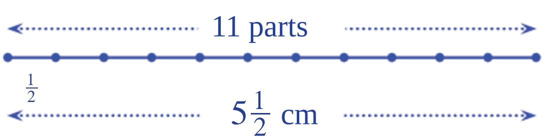
What is the area of a rectangle of length $5\frac{1}{2}$ centimetres and breadth $3\frac{1}{3}$ centimetres ?



Into how many parts of length $\frac{1}{2}$ centimetre each, can we divide the bottom side?

10 lines of length $\frac{1}{2}$ centimetre, each make 5 centimetres. To make $5\frac{1}{2}$ centimetres, we need one more such line

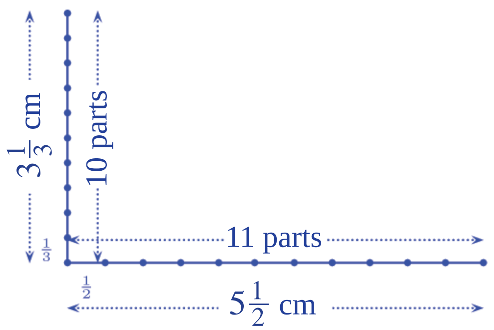
$$5\frac{1}{2} = 11 \times \frac{1}{2}$$



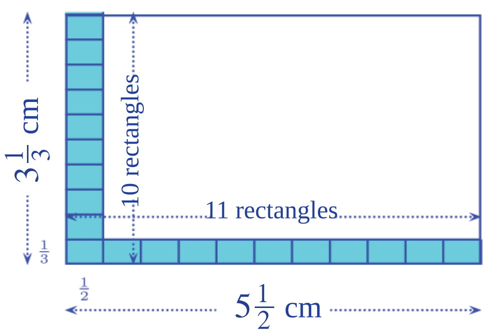
Now let's divide the left side of the rectangle into parts of length $\frac{1}{3}$ centimetre.

9 lines of length $\frac{1}{3}$ centimetre each, make 3 centimetres. To get $3\frac{1}{3}$ centimetres, we need one more line.

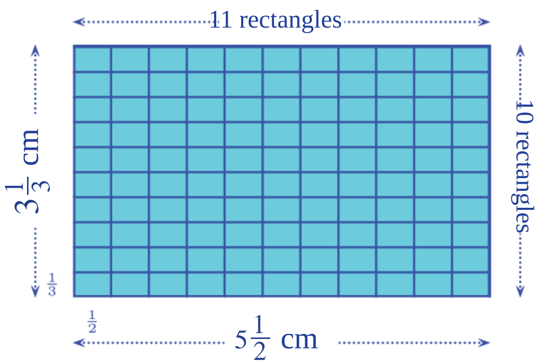
$$3\frac{1}{3} = 10 \times \frac{1}{3}$$



We now fill part of the rectangle with small rectangles of length $\frac{1}{2}$ centimetre and breadth $\frac{1}{3}$ centimetre.



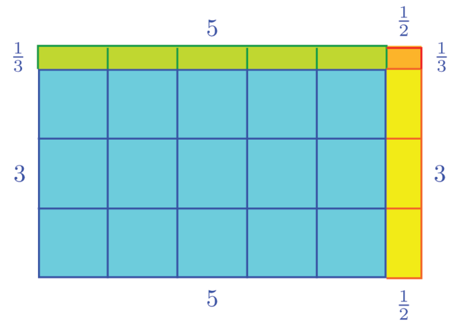
How many such small rectangles are needed to fill the rectangle completely ?



$11 \times 10 = 110$ small rectangles in all; each of area $\frac{1}{6}$ square centimetre.

Rectangle Division

We can divide a rectangle of length $5\frac{1}{2}$ cm and breadth $3\frac{1}{3}$ cm this way also :



The area can be calculated in four steps :

Blue rectangle

$$5 \times 3 = 15$$

Green rectangle

$$5 \times \frac{1}{3} = 1\frac{2}{3}$$

Yellow rectangle

$$\frac{1}{2} \times 3 = 1\frac{1}{2}$$

Red rectangle

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Total area

$$15 + 1\frac{2}{3} + 1\frac{1}{2} + \frac{1}{6} = 18\frac{1}{3} \text{ sq.cm}$$

Total area.

$$110 \times \frac{1}{6} = 18\frac{1}{3} \text{ square centimetres}$$

Let's take another look at the operations involved :

$$5\frac{1}{2} = 11 \times \frac{1}{2}$$

$$3\frac{1}{3} = 10 \times \frac{1}{3}$$

$$11 \times 10 \times \frac{1}{2} \times \frac{1}{3} = 110 \times \frac{1}{6}$$

The last multiplication may also be written as,

$$11 \times 10 \times \frac{1}{2} \times \frac{1}{3} = \left(11 \times \frac{1}{2}\right) \times \left(10 \times \frac{1}{3}\right)$$

$$= 5\frac{1}{2} \times 3\frac{1}{3}$$

Thus we see that even if the lengths are in fractions, the area of a rectangle is still the product of the lengths of sides.

Now try these problems :

(1) The length and breadth of some rectangles are given below. Find the area of each:

(i) $3\frac{1}{4}$ centimetres, $4\frac{1}{2}$ centimetres

(ii) $5\frac{1}{3}$ metres, $6\frac{3}{4}$ metres

(iii) $1\frac{1}{3}$ metres, $\frac{3}{4}$ metres

(2) What is the area of a square of side $1\frac{1}{2}$ metres ?

(3) The perimeter of a square is 14 metres. What is its area ?

3 TRIANGLES

Star Picture

Look at this picture:

How do we draw this figure ?

What all do we need to draw ?

Two triangles and a circle.

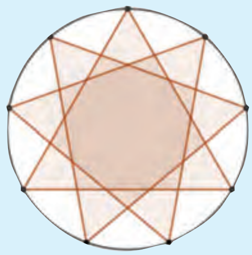
What is special about the triangles ?

All sides are equal.

Let's draw such a triangle first. Let's take the sides to be 3 cm long.

First we draw the bottom line.

Let's see how to draw stars using geogebra. Draw a regular hexagon using the **Regular Polygon** tool. (For this, choose the tool and click on the two points. In the pop-up window give the number of corners as 6). Join alternate corners to get an equilateral triangle. Joining the other three corners, we can draw the next triangle also. To draw the circles, use the **Circle through 3 points** tool and click on three corners. Now we may hide the hexagon. Like this we can draw a regular polygon of 9 sides and draw this star:



Here we have three equilateral triangles. Try drawing star pictures with more equilateral triangles.

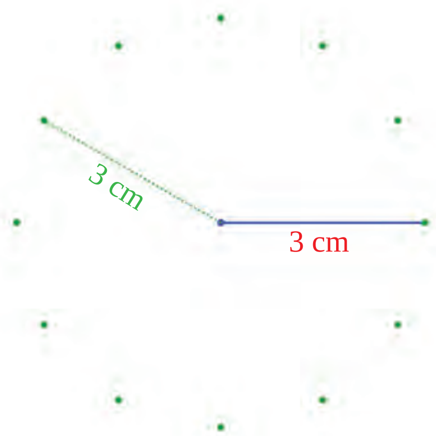
Where do we mark the third corner of the triangle?

The left and right sides need to be 3 centimetres each.

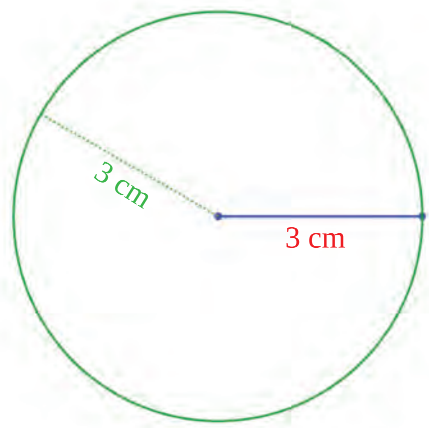
So the third corner must be 3 centimetres away from the two ends of the first line.

How many points can you mark 3 centimetres away from one end ?

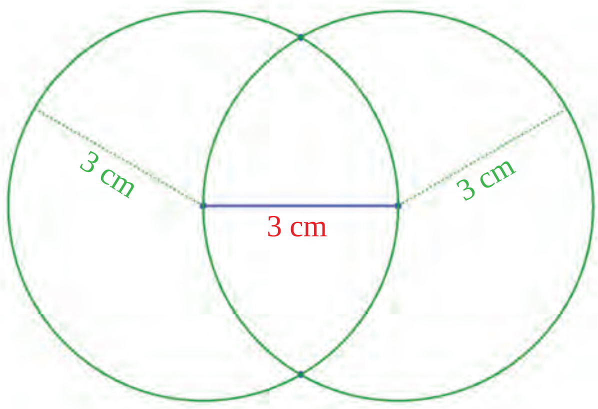
Imagine such points :



If we draw this circle, the third corner must be somewhere on it.



Like this, if we draw a circle of radius 3 centimetres, centred on the right end, the third corner must be somewhere on this circle also.



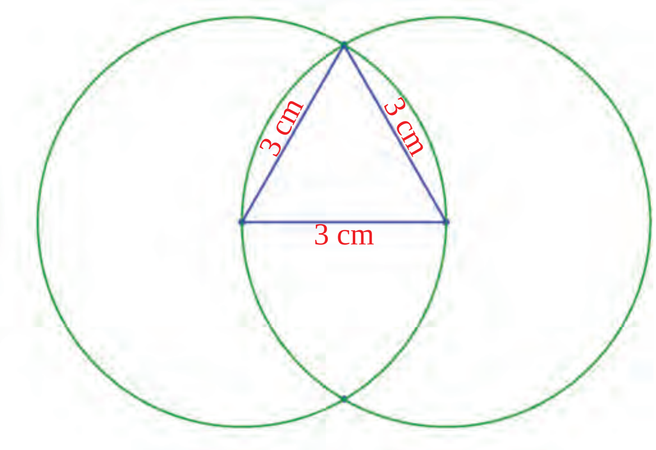
These circles intersect at two points at the top and bottom, don't they ?

What can you say about those points ?

Since they are on the first circle, they are 3 centimetres away from the left end. Also as they are on the second circle, they are 3 centimetres away from the right end.

Any one of these could be the third corner of our triangle.

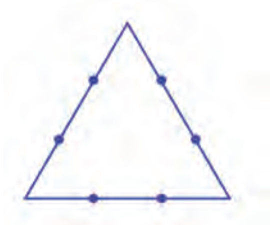
Let's draw the triangle using the top point.



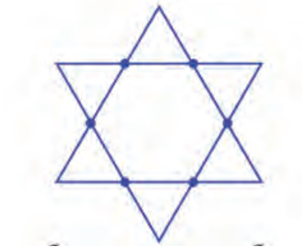
Now we can delete the circles.

We have one triangle now. How do we draw the star ?

For this, mark points on the three sides 1 centimetre apart.



Joining these points, we can draw the second triangle.

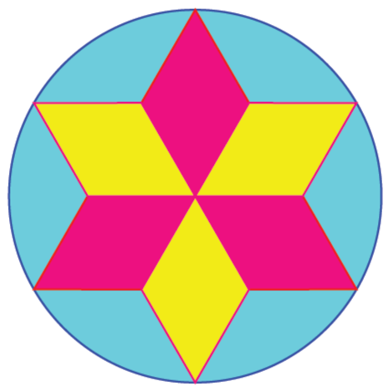


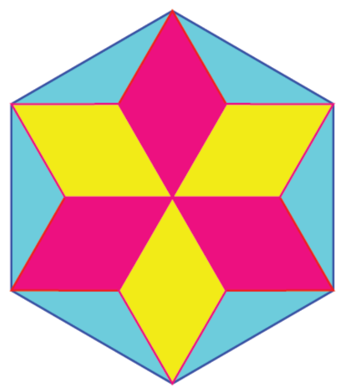
If we draw a circle around it, we have the first figure. Where do we fix the compass to draw it ?

Find it out (Wild guesses won't do, it should be exact).

When the picture is completed, you can colour it as you wish and make it pretty.

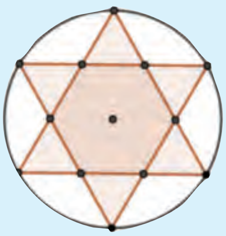
By drawing two triangles like this, adding some lines, removing some and with a change of colour, you can get the pictures below:



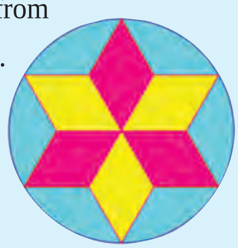


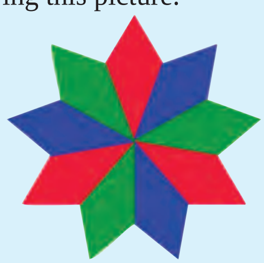
Try it out.

We have drawn a star using the regular hexagon. We can draw other beautiful pictures like this. Using the **Intersect** tool, mark the points where the sides of the triangle meet. Use the midpoint of centre tool to find the centre of the circle.

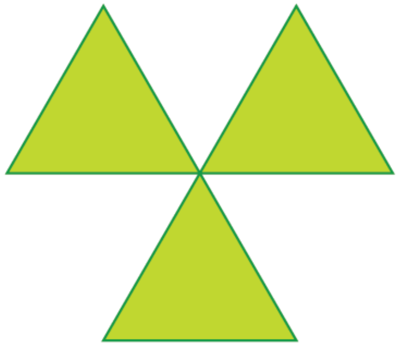


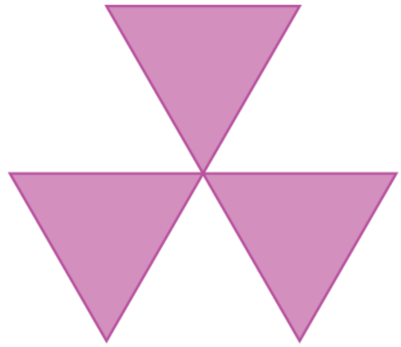
Now hide the triangle and use the polygon tool to join the dots properly to draw various figures. Right click on the polygon and choose the appropriate colour from **Object properties → colour**. Use **Opacity** to change the clarity of the picture. Try drawing this picture.





Also try to draw the pictures below :

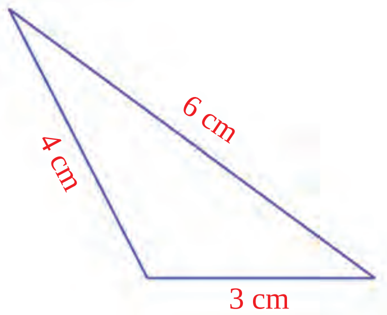




Lines and Math

We've drawn triangles with all sides equal. Such triangles are called *equilateral triangles*. Can't we draw triangles with unequal sides also this way ?

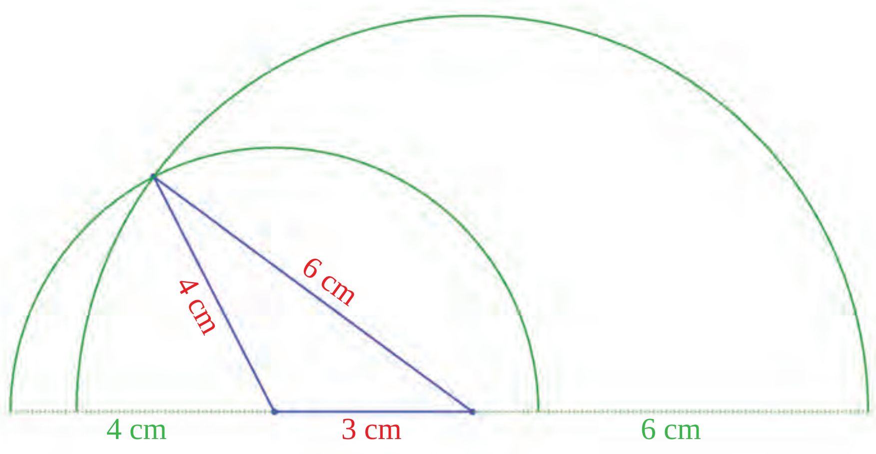
For example, a triangle with sides 3 centimetres, 4 centimetres and 6 centimetres.



How do we draw this ?

Draw a line 3 centimetres long. Mark the third corner, 4 centimetres from one end and 6 centimetres from the other end.

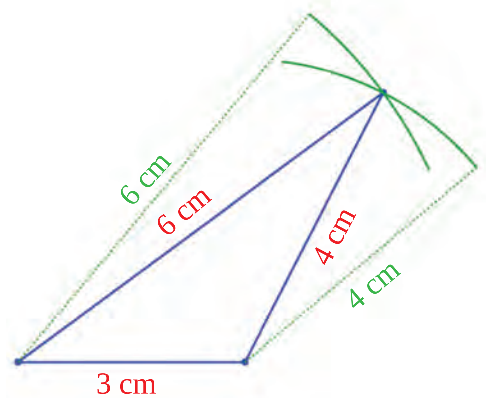
For this, isn't it enough to draw a circle from each end ?



For the third corner, we need only the position above. So we draw only the top half of the circles (Even that is not needed, in fact. It's enough to draw two small pieces of the circles).

> **Two Sides**
>
> Draw triangle *ABC*, with *AB* = 6, *AC* = 5. Use **Segment with Given Length** tool to draw a line *AB* of length 6. Draw a circle of radius 5 centimetres centred at *A* and mark a point C. Draw triangle *ABC*. Try changing the position of *C* along the circle. Do the lengths of the sides *AB*; *AC* change? What about the length of *BC* ? What is the maximum length ? What is the minimum length ? What is the relation between that of the possible length of this side with the other two sides.

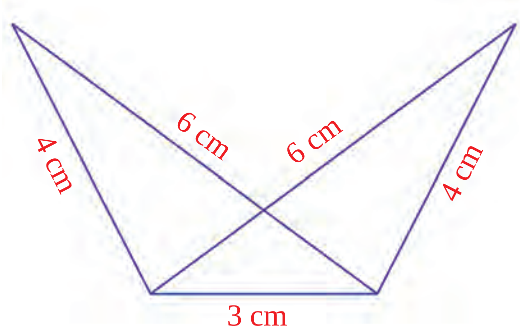
We can also draw like this, interchanging the sides.



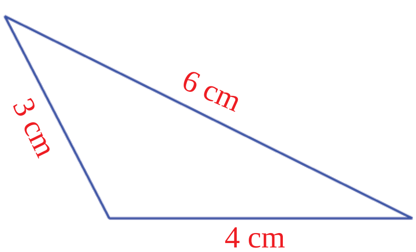
Is there any difference between these two triangles ?

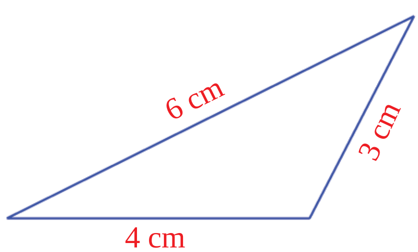
Just flipped right to left, isn't it ?

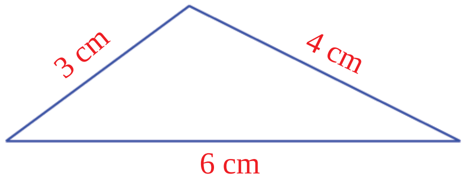
We can see this immediately if we draw them together.

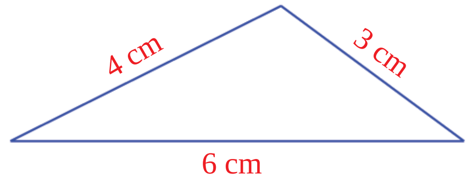


Without changing the lengths of the sides, we can draw the triangles changing the position of the sides.





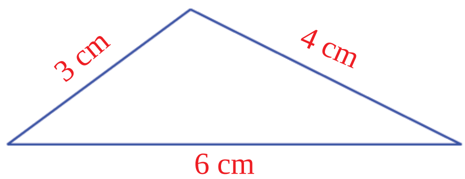


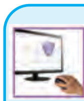


Apart from flips and turns, is there any real difference between these six triangles ?

If you have any doubt, cut out one of these figures from a thick piece of paper and place it on each in different ways and see.

Look at one of those figures now.





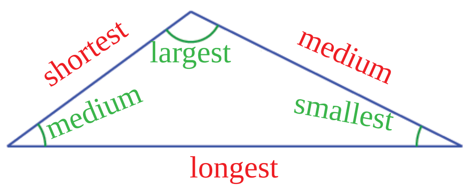
Draw a triangle and mark the measures of angles and sides. Compare the size of the angles and the length of the sides. Change the vertices of the triangle and see what happens.

Which is the largest angle ?

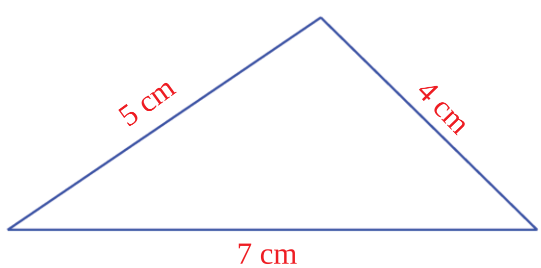
The smallest ?

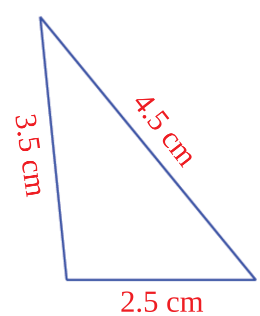
Can't you see it without actually measuring?

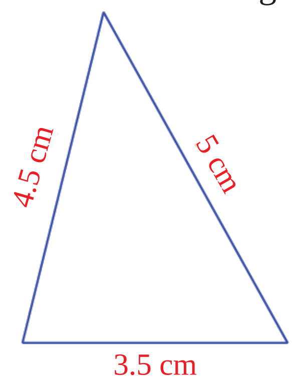
Look at the sides opposite them :



Draw several triangles with different lengths for the sides. In all these, is the longest side is opposite the largest angle, and the shortest side opposite the smallest angle ?







Why is it so ?

As the angle becomes larger, its sides spread out more, don't they ?

This fact about triangles can be stated thus:

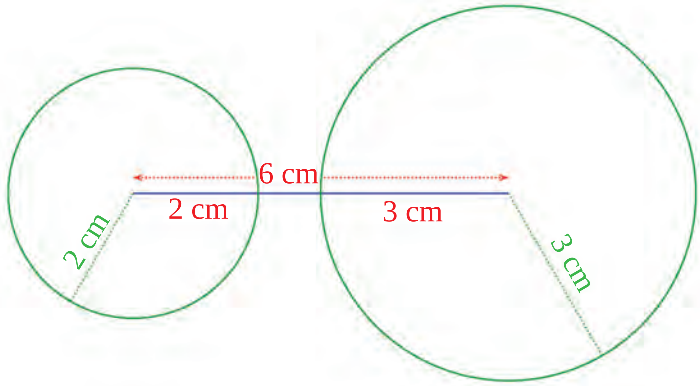
In any triangle, the angles and their opposite sides have sizes in the same order.

We have drawn triangles with different sides.

Draw a line AB, 6 units long. Draw a circle with centre A and radius 2 units. Make a slider a with Min = 0, Max = 10. Choose **Circle → Centre & Radius** tool and click on B. Give the radius of the circle as a in the window. Use **Intersect** tool to mark the point C of the intersection of the circles. (The value on the slider can be adjusted so that the circles intersect). Draw triangle ABC and mark the lengths of the sides. Change the radius using the slider and check. For what values do you get a triangle ?

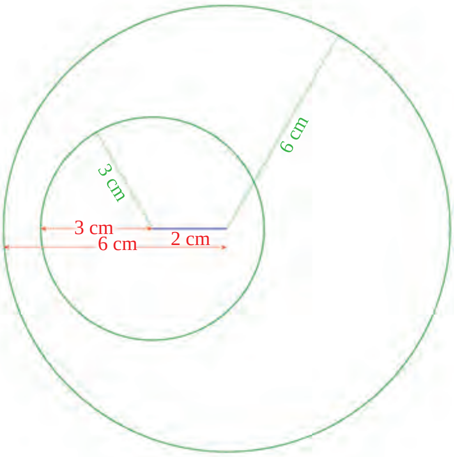
This raises a question. Can we draw a triangle with any three numbers as the lengths of the sides?

For example, let's try, 6 centimetres, 2 centimetres, 3 centimetres.



We see that all the points 2 centimetres away from one end of the 6 centimetre long line, are more than 3 centimetres away from the other end; and the other way round also.

What if we try to draw the triangle with the 2 centimetre long line as the bottom side?



Take another look at the red lines in the two pictures. Why is it that we can't draw a triangle ?

The greatest length 6 is more than the sum of the other two lengths 2 and 3.

So, for three lengths to be the sides of a triangle, the greatest length must be less than the sum of the other two.

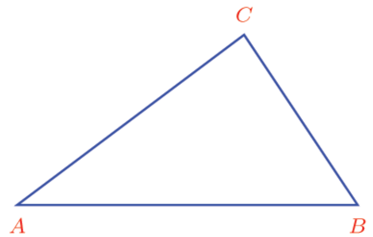
This we state as a general principle about the lengths of the sides of a triangle:

In any triangle, the length of the greatest side is less than the sum of the length of the other two sides.

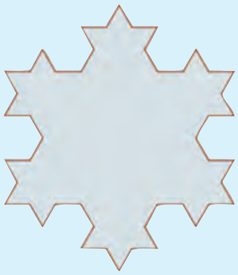
This may be worded differently. *The sum of the length of the two smaller sides of a triangle is more than the length of the greatest side*.

From this, we can see that the sum of the lengths of any two sides of a triangle is greater than the length of the third side (Why is it so ?).

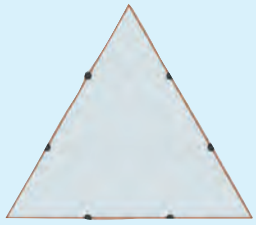
This is easily seen with a little thought.



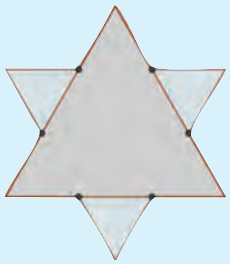
The direct path from A to B is definitely shorter than a detour through C, however near C is to A, isn't it?



Let's draw a picture as shown above. For this, we first draw an equilateral triangle, and divide each side into three equal parts.



Draw equilateral triangles on the middle part of each side.



Next divide each side of this star into three equal parts and draw equilateral triangles. Now our picture is ready. If the sides of the first triangle is a multiple of 9 then, things are easy. To divide a side into three equal parts draw a circle of radius one third the side, centred at a corner of the triangle and mark the point where the circle meets the side.

Try these problems:

(1) The sides of a triangle are natural numbers. If the lengths of two sides are 5 centimetres and 8 centimetres, what are the possible numbers which can be the length of the third side ?

(2) The lengths of the sides of a triangle are all natural numbers and two of the sides are 1 centimetre and 99 centimetres. What is the length of the third side?

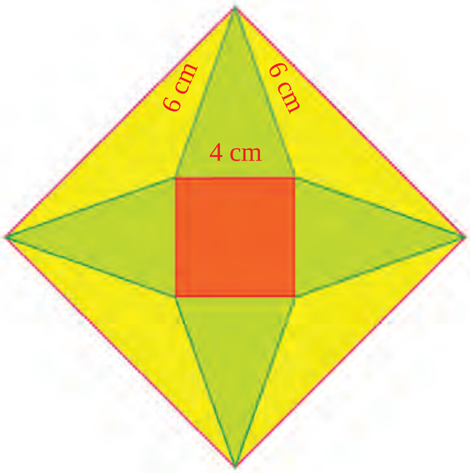
(3) Which of the following sets of three lengths can be used to draw a triangle?

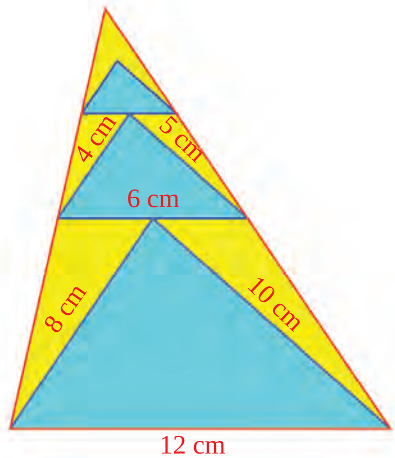
(i) 4 centimetres, 6 centimetres, 10 centimetres

(ii) 3 centimetres, 4 centimetres, 5 centimetres

(iii) 10 centimetres, 5 centimetres, 4 centimetres

(4) Draw these pictures :





Angle math

We've seen how to draw triangles, with specified lengths for the sides. We also found out the relation between the lengths of the sides.

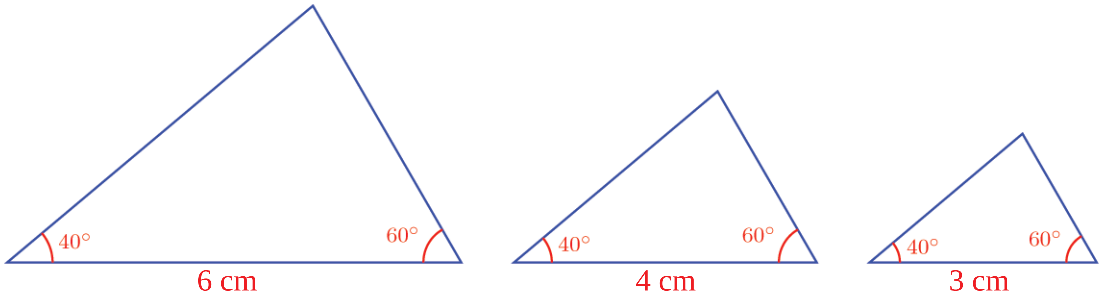
What if we specify the angles?

We know that the sum of the angles of a triangle is 180°(The chapter, **Parallel Lines**).

So we can fix only two angles. For example, if we take two angles to be 40° and 60°, then the third angle has to be 80°.

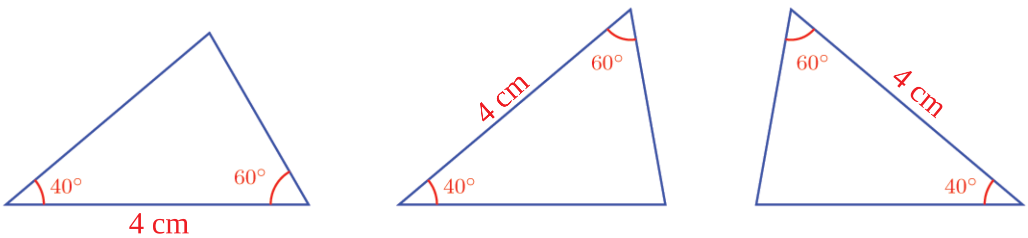
How do we draw the triangle with these angles?

We can draw not one, but several triangles, right?

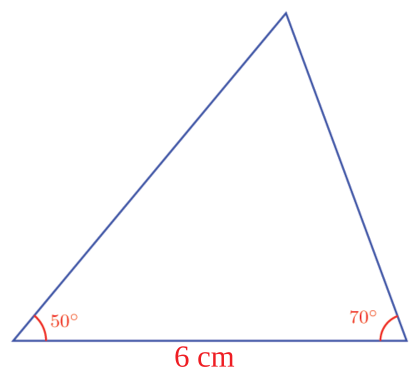


So, if we have one particular triangle in mind, we must specify not only two angles, but the length of the side on which they stand also.

This raises a question : can't we draw several triangles with one side of length 4 centimetres and two angles of measures 40° and 60° standing on it?

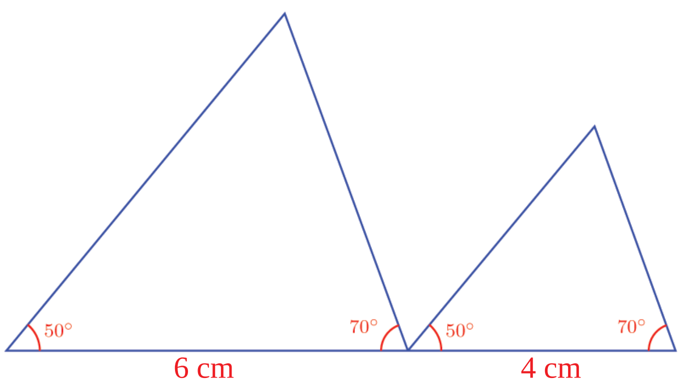


We may interchange the position of the angles and draw several more (Try it!). But aren't they all, the same triangle flipped and turned, as we saw earlier? That is to say, there is in fact only one triangle with one side 4 centimetres and angles 40°, 60° on this side.

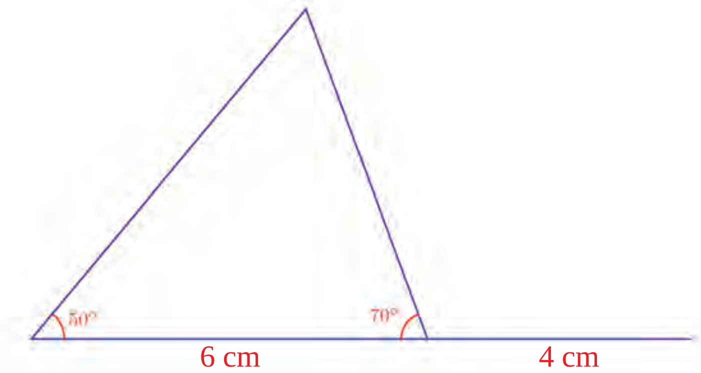


It is easy to change the size of a triangle with these angles. For example, look at this figure.

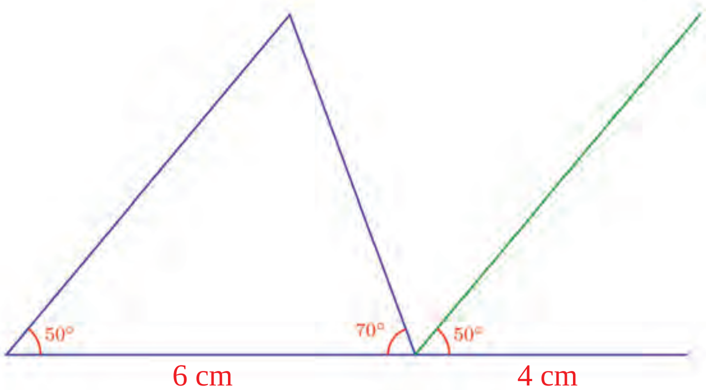
We want to draw another triangle adjacent to it as shown below :



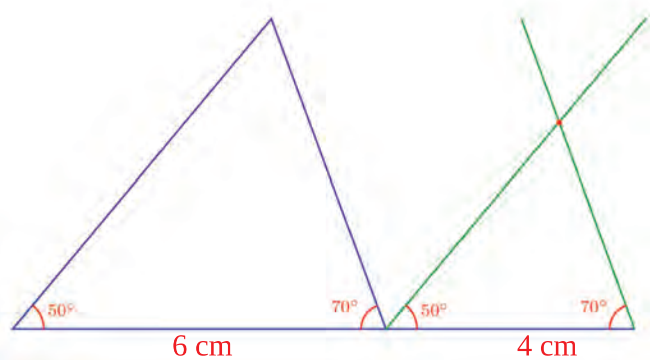
First we extend the bottom side by another 4 centimetres.



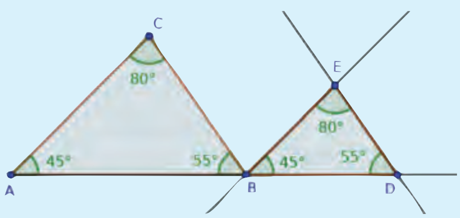
We make a 50° angle at the left end of the extended side .



And a 70° angle at the right end of the line.



Draw a triangle *ABC* using the **Polygon** tool. Use the **Ray** tool to extend *AB*. Mark a point *D* on *AB*. Through *D*, draw a line parallel to *BC* and another line through *B* parallel to *AC*. Mark *E*, their point of intersection. Draw triangle *BDE*. Choose the **Angle** tool and click inside the triangles to see the angle measures.



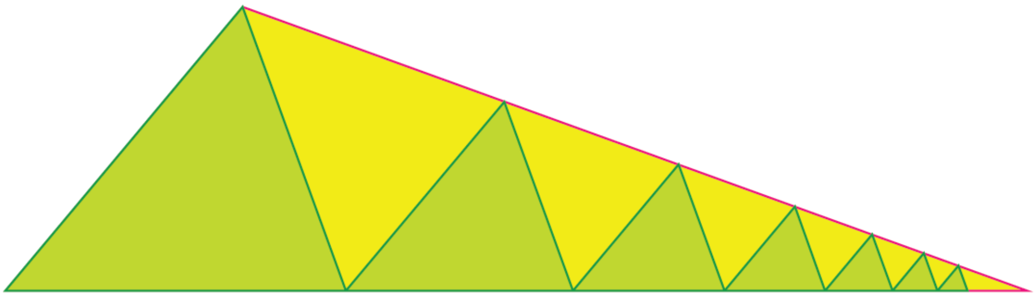
Change the position of *D* along the line and see what happens.

Did you notice something about the figure ?

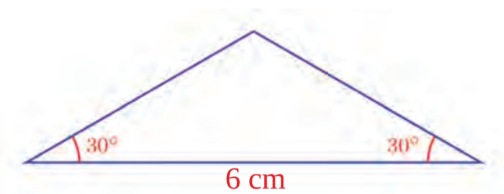
The sides of the new triangle are parallel to the sides of the first triangle (Why ?).

So, instead of drawing a new triangle, by measuring the angles once again, it is enough to draw lines parallel to the sides of the first triangle (This is easy in **GeoGebra**)

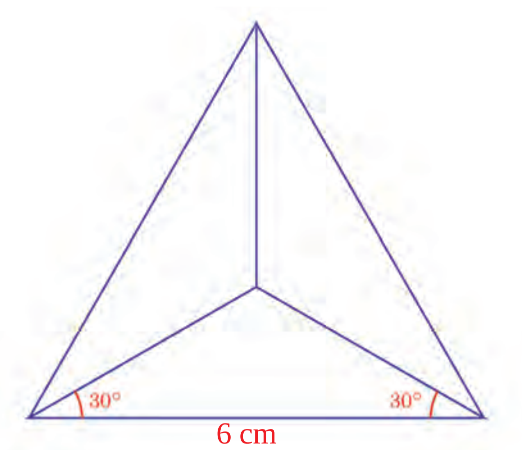
Now, can't you draw a picture like the one below?



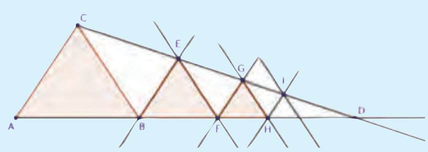
Can you draw the triangle shown below ?



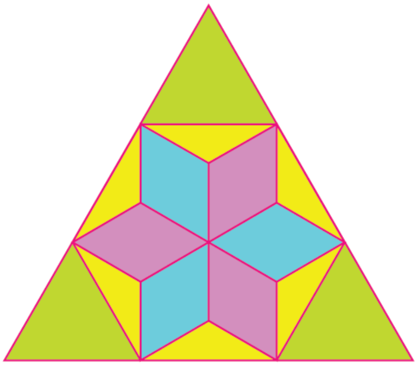
By joining three of them we have an equilateral triangle.

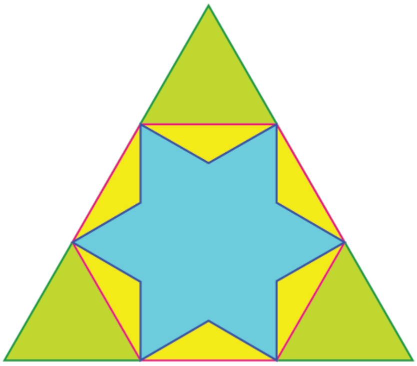



Draw triangle *ABC*. Extend *AB* and mark *D* on it. Join *CD*. Through *B*, draw a line parallel to *AC*. Mark *E*, the point of intersection of this line with *CD*. Draw a line through *E* parallel to *BC*. Mark *F*, the point of intersection of this line with the bottom line. Draw triangle *BFE*. Draw other triangles similarly and colour them. Now we can hide the unnecessary lines.



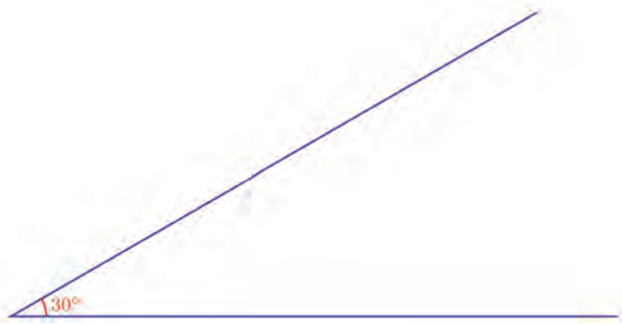
Can you draw these pictures by joining more and more small triangles like this ?



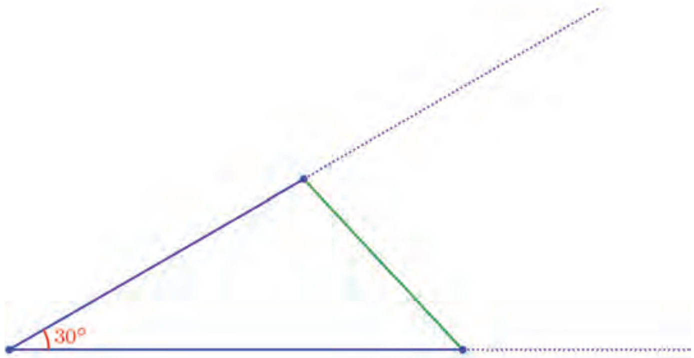


Sides and angles

The sides of an angle may be extended as much as we want :



If we block them with a line, we have a triangle.

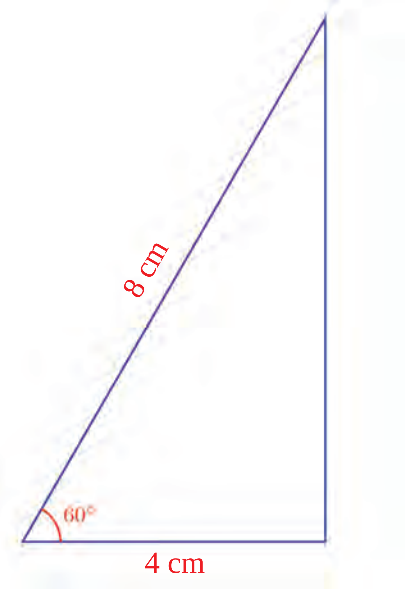


To draw such a line, it is enough to mark a point on each of the sides, right ?

In other words, once we specify an angle and the length of its sides, we have the triangle.

For example, if we fix the bottom side as 6 centimetres and top side as 4.5 centimetres of the above angle, we get the earlier triangle. This can be put in another way: once if we specify the lengths of two sides of a triangle and the angle between them, we have the triangle.

Now can't you draw a triangle with two sides 4 centimetres and 8 centimetres and the angle between them as 60° ?



Do you note anything special about this triangle ?

Draw other triangles with one side twice the other side and the angle between them as 60°.

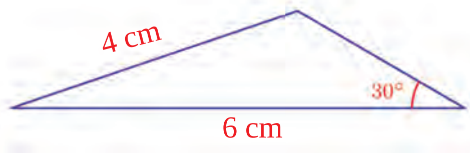
Are all of them right triangles ?

Now what is the third angle of these triangles ?

From the set squares in the geometry box, choose the one with all sides different and measure the shortest and longest sides. Is the longest side twice the shortest ?

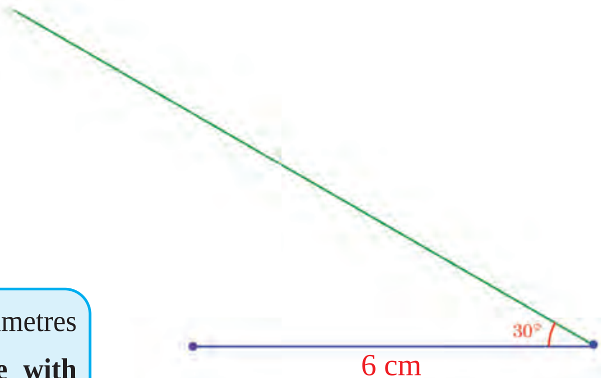
Let's now look at another thing. Once we specify two sides and the angle between them, we can draw the triangle.

Suppose we specify the lengths of two sides and an angle not between these sides.

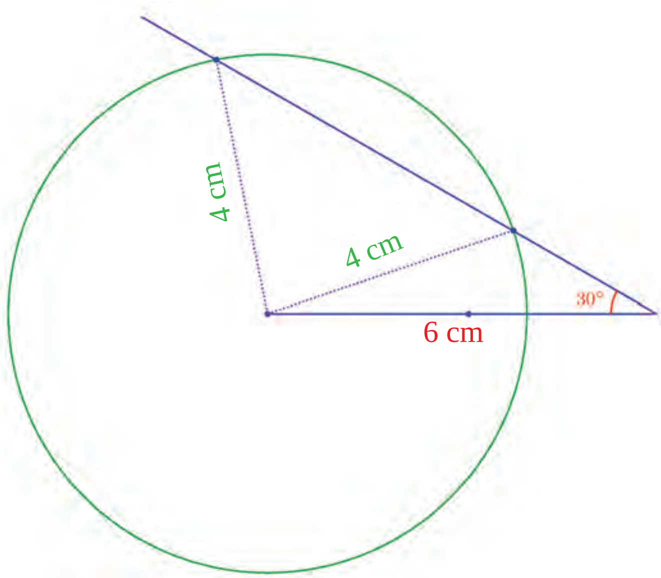


How do we draw this ?

We can draw a line 6 centimetres long and an angle of 30° at the right end:

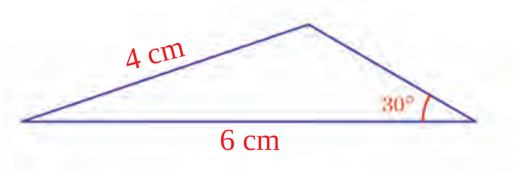


The third corner of the triangle could be anywhere on this line; but it should be 4 centimetres away from the left end of the bottom line. That is to say, it should be on a circle of radius 4 centimetres with the centre as the left end. Let's draw this circle.

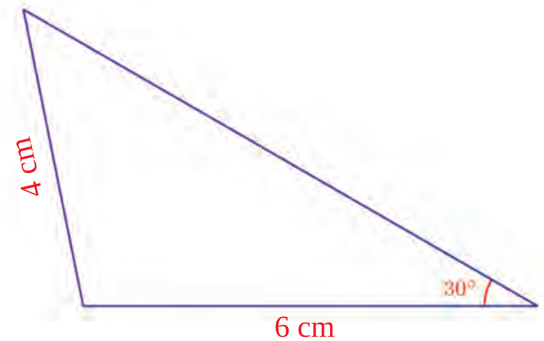


Draw a line *AB*, 10 centimetres long. Using the **Angle with Given Size** tool, click on *B* and then on *A* to get a window. Give 30° as the angle measure, we get a point *B′*. Draw line *AB′* (Use **Ray** tool). Make a slider '*a*'. Choose **Circle → Centre & Radius** tool click on *B*. In the window give a as the radius of the circle. Mark the points of intersections *C*, *D* of the line *AB′* with the circle. Draw triangles *ABC*, *ABD*. Try changing the value of the slider. When do we get more than one triangle ? When we get a single triangle do you see anything special about it ? Is, there a case when we don't get a triangle at all ?

We see that the circle cuts the line at two points. If we take the bottom point as the third corner, then we get triangle we have seen above.



What if we take the top point as the third corner ?

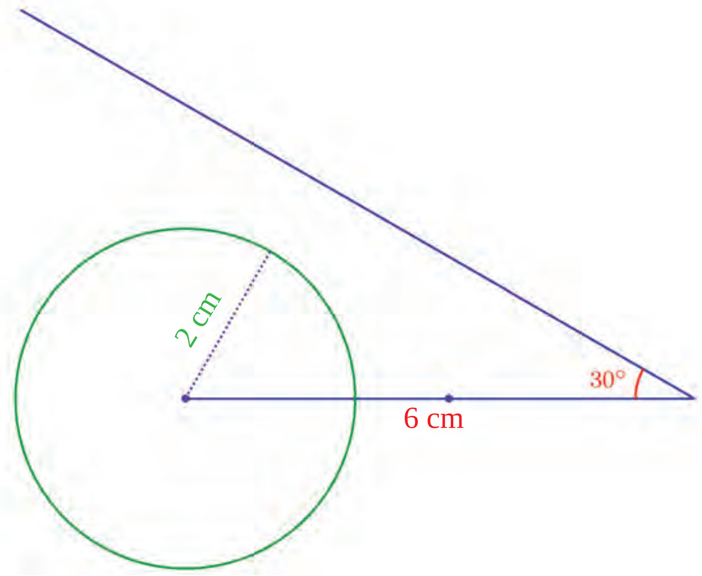


Thus, we can draw two triangles with these measures.

Do we get two triangles, if we take the length of the left side as something other than 4 centimetres?

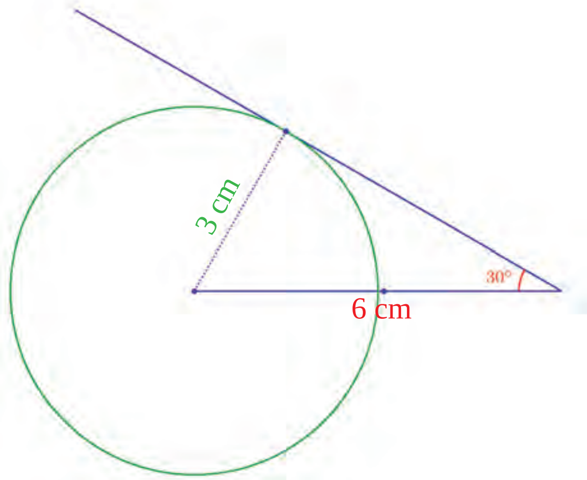
For example, what if it is 2 centimetres ?

Draw the circle and the line as below :



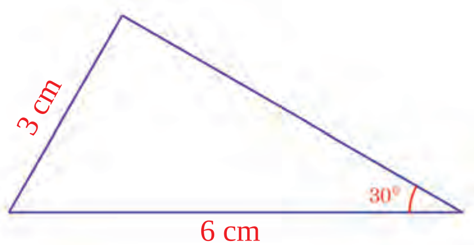
Circle doesn't intersect the line; and we cannot draw a triangle.

What if the length of the side is 3 centimetres ?



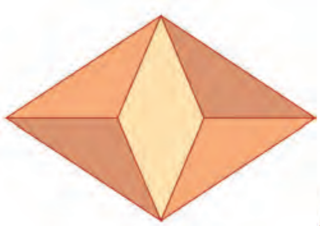
Circle intersects the line at a single point.

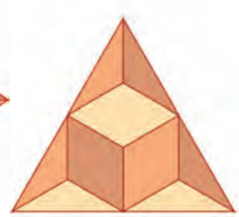
And we can draw a single triangle with these measures.

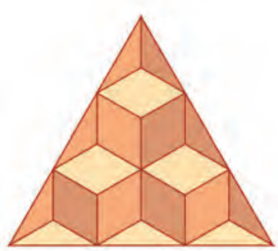


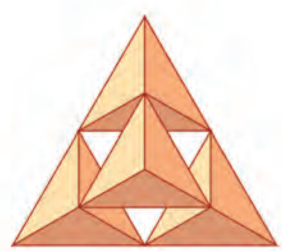
What happens if we take the side longer than 6 centimetres ? Try it!

Now, draw the following figures using the methods we have used in this lesson.

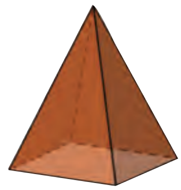




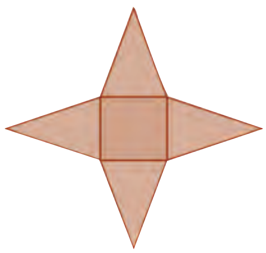




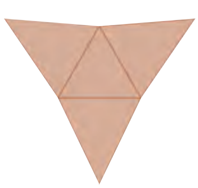
3 D Shapes

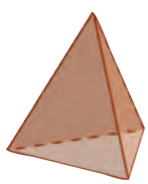


This is the picture of a square pyramid. Can you construct similar one? Look the shape below. On each side of the square, there is an isosceles triangle drawn (triangle with two sides equal).



Cut a similar one from a thick sheet of paper. When we fold and paste them we get a square pyramid.





If we draw isosceles triangles on each side of an equilateral triangle instead, as above, then we get a pyramid with an equilateral triangle as base.

4 RECIPROCALS

Times and parts

There are two pens in one packet and eight in another:

We can compare these in two ways:

Now look at these weights:

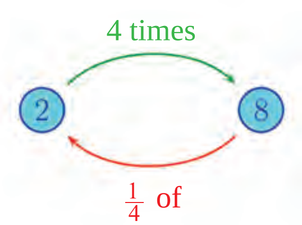
One is 5 kilograms, the other 15 kilograms.

How do we say it as times and part ?

Let's look at the first example using only numbers.

4 times 2 is 8

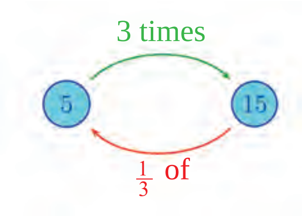
$\frac{1}{4}$ of 8 is 2



Let's do this for the second example also:

3 times 5 is 15

$\frac{1}{3}$ of 15 is 5



Now try this problem :

A vessel holds 10 litres of water. Another vessel holds 15 litres. In what all ways can you state the relation between them as times and part ?

Let's think this way. To fill the large vessel, how many times should we fill and empty the small vessel ?

When we do this once, we have 10 litres in the large vessel. Now only 5 more litres is needed. That means $\frac{1}{2}$ of the small vessel.

Thus, 15 litres is $1\frac{1}{2}$ times 10 litres.

Let's think in the reverse.

If we take the 15 litre vessel full of water, not all of it is needed to fill the 10 litre vessel. How much water will be left in the large vessel ?

The remaining 5 litres is $\frac{1}{3}$ of 15 litres, isn't it ? So the 10 litres poured into the small vessel is $\frac{2}{3}$ of 15 litres.

That is,

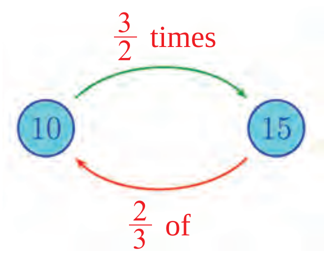
$1\frac{1}{2}$ times 10 litres is 15 litres

$\frac{2}{3}$ of 15 litres is 10 litres

In terms of numbers alone

$\frac{3}{2}$ times 10 is 15

$\frac{2}{3}$ of 15 is 10



We can think of it in a different way.

If we take $\frac{1}{10}$ of 10, we have 1; and 15 times 1 is 15.

So 15 is $\frac{15}{10}$ times 10.

$$\frac{15}{10} = \frac{3 \times 5}{2 \times 5} = \frac{3}{2}\text{, isn't it ?}$$

We can think of the reverse problem also like this

$\frac{1}{15}$ of 15 is 1; and 10 times 1 is 10. So $\frac{10}{15}$ of 15 is 10.

$$\frac{10}{15} = \frac{2 \times 5}{3 \times 5} = \frac{2}{3}$$

Now can you do this problem ?

The length of a rope is 4 metres. The length of another rope is 14 metres. How do we state the relation between them as times and part ?

$\frac{14}{4}$ times 4 is 14

$\frac{4}{14}$ of 14 is 4

Let's rewrite the fractions,

$$\frac{14}{4} = \frac{7 \times 2}{2 \times 2} = \frac{7}{2}$$

$$\frac{4}{14} = \frac{2 \times 2}{7 \times 2} = \frac{2}{7}$$

Thus,

$\frac{7}{2}$ times 4 is 14

$\frac{2}{7}$ of 14 is 4

We can rewrite $\frac{7}{2}$ in the first line as $3\frac{1}{2}$ and say this way:

$3\frac{1}{2}$ times 4 is 14

$\frac{2}{7}$ of 14 is 4

Now try these problems:

(1) Suma has 16 rupees with her. Safeer has 4 rupees.

   (i) What part of Suma's money does Safeer have ?

   (ii) How many times Safeer's money does Suma have ?

(2) A large bag contains 9 kilograms of sugar. A small bag contains 6 kilograms.

   (i) The weight of sugar in the heavier bag is how much times that in the lighter bag ?

   (ii) The weight of sugar in the lighter bag is what part of that in the heavier bag ?

(3) The weight of an iron block is 6 kilograms. The weight of another block is 26 kilograms.

   (i) The weight of the lighter block is what fraction of that of the heavier block ?

   (ii) The weight of the heavier block is how much times that of the lighter block ?

(4) The length of a ribbon is $2\frac{2}{3}$ times the length of a smaller ribbon. What part of the length of the large ribbon is the length of the small ribbon ?

Topsy-turvy

Did you notice something in the problems we did just now ?

In order to reverse times and part, we just need to turn fractions upside down. Can you say this in the first examples we discussed ?

For example, in the pen problem, we saw

$$4 \text{ times } 2 \text{ is } 8$$
$$\frac{1}{4} \text{ of } 8 \text{ is } 2$$

If we take 4 as $\frac{4}{1}$, then we are turning the fractions here too.

Instead of saying, "turning the fraction upside down", we can also say (more formally), "interchanging the numerator and denominator". The fractions obtained like this are called *reciprocals.*

For example,

- $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals of each other;
- $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals of each other

Now try this problem:

The price of $1\frac{1}{2}$ *kilograms of tomato is 30 rupees. What is the price of one kilogram tomato* ?

We may think this in several ways. One way is this:

- 3 times $\frac{1}{2}$ is $1\frac{1}{2}$
- 30 rupees is 3 times the price of $\frac{1}{2}$ kilogram of tomato
- The price of $\frac{1}{2}$ kilogram is $30 \div 3 = 10$ rupees
- So, price of 1 kilogram is $10 \times 2 = 20$ rupees

Another way ;

- 2 times $1\frac{1}{2}$ is 3
- The price of 3 kilograms of tomato is $30 \times 2 = 60$ rupees
- Price of 1 kilogram is $60 \div 3 = 20$ rupees

We can work it out directly using reciprocals:

- 30 rupees is $1\frac{1}{2}$ times the price of one kilogram of tomato.

- $1\frac{1}{2}$ is $\frac{3}{2}$
- The price of one kilogram is $\frac{2}{3}$ of 30 rupees.
- $\frac{2}{3} \times 30 = 20$ rupees

Now let's try another problem :

The area of a rectangle is $\frac{1}{2}$ square metre and the length of one side is $\frac{3}{4}$ metre; What is the length of the other side ?

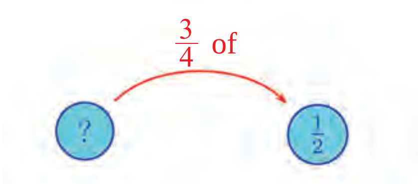
We've seen that the area of a rectangle is the product of the lengths of its sides, even if they are fractions. So the number $\frac{1}{2}$, which is the area in this problem, is the product of $\frac{3}{4}$, which is the length of one side, and the number giving length of the other side,.

Thinking it out in terms of numbers alone, the problem is :

What number multiplied by $\frac{3}{4}$ gives $\frac{1}{2}$?

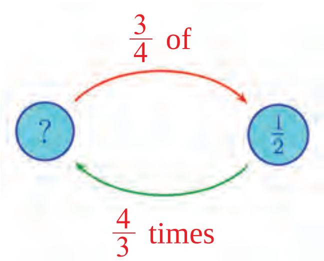
Multiplying by $\frac{3}{4}$ means taking $\frac{3}{4}$ part. So the question becomes.

$\frac{3}{4}$ of what number is $\frac{1}{2}$?



We can use reciprocals to reverse this.

The number we need is $\frac{4}{3}$ times $\frac{1}{2}$.



$$\frac{4}{3} \times \frac{1}{2} = \frac{2}{3}$$

That is, the length of other side of the rectangle is $\frac{2}{3}$ metre.

Now try these problems:

(1) 27 students of a class got A plus in Maths. They form $\frac{3}{4}$ of the entire class. How many students are there in the class ?

(2) $\frac{2}{3}$ of a bottle was filled with $\frac{1}{2}$ litre of water. How many litres of water will the bottle hold ?

(3) $\frac{3}{4}$ of a vessel holds $1\frac{1}{2}$ litres of water. What is the capacity of the vessel in litres if it is completely filled with water ?

(4) Two of the three ribbons of the same length and half the third ribbon were placed end to end. It came to 1 metre. What is the length of a ribbon in centimetres ?

Fraction division

The price of 5 pens is 40 rupees. What is the price of a pen ?

We need to divide 40 by 5, right ?

$$40 \div 5 = 8$$

The price of a pen is 8 rupees.

Isn't division by 5 the same as computing a part ?

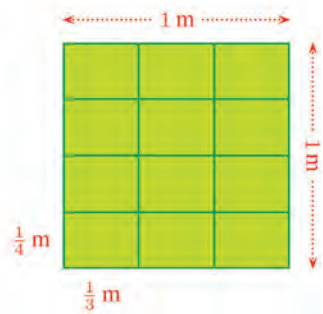
$$\frac{1}{5} \times 40 = \frac{40}{5} = 8$$

In other words, division of natural numbers is the same as multiplication by the reciprocal.

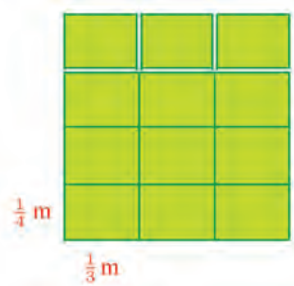
So, for fractions also, multiplication by reciprocal can be called division; and written as an operation of division.

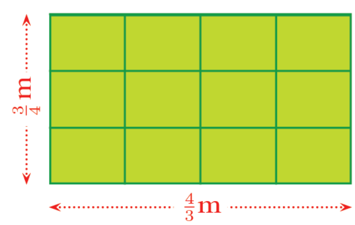
Reciprocal Multiplication

A square of side 1 metre is divided into equal pieces as shown in the figure:



The three small squares in the top row are cut and placed to the left of the rectangle like this.





The area of this rectangle is $\frac{4}{3} \times \frac{3}{4}$ square metre.

We've not removed anything from the first square; nor have we added anything to it. So the area should be the same.

That is, $\frac{4}{3} \times \frac{3}{4} = 1$

For example, in the rectangle problem, done earlier, we can say that to find the length of the second side we have to divide $\frac{1}{2}$ by $\frac{3}{4}$:

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$$

Now do the following problems using reciprocals, and explain them as division problems:

(1) A 16 metres long rod is cut into pieces of length $\frac{2}{3}$ metre. How many such pieces will be there ?

(2) How many $\frac{3}{4}$ litre bottles are needed to fill $5\frac{1}{4}$ litres of water ?

(3) $13\frac{1}{2}$ kilograms of sugar is to be packed into bags with $2\frac{1}{4}$ kilograms sugar each. How many bags are needed ?

(4) The area of a rectangle is $22\frac{1}{2}$ square centimetres and one side is $3\frac{3}{4}$ centimetres long. What is the length of the other side ?

(5) How many pieces, each of length $2\frac{1}{2}$ metres, can be cut off from a rope of length $11\frac{1}{2}$ metres ? How many metres of rope will be left ?

A Problem

A rod is 36 metres long. How many pieces each of length $2\frac{1}{2}$ metres can be cut off from it ? What is the length of the rod left ?

Appu did the problem this way.

$$36 \div 2\frac{1}{2} = 36 \div \frac{5}{2}$$
$$= 36 \times \frac{2}{5}$$
$$= \frac{72}{5}$$

When we divide 72 by 5, the quotient is 14 and remainder is 2. So we get 14 pieces. The remaining rod is of length 2 metres.

Ammu used another idea.

2 pieces, each of length $2\frac{1}{2}$ metres make 5 metres

$$7 \times 5 = 35$$

So $7 \times 2 = 14$ pieces can be cut off. The remainder is $36 - 35 = 1$ metre

Whose answer is right ?

5 DECIMAL METHODS

Decimal forms

Let's start with a small problem :

A wire of length 43 centimetres is cut into ten equal pieces. What is the length of each piece ?

We can quickly say $\frac{43}{10}$ centimetres. We can explain a little more, by saying it is $4\frac{3}{10}$ centimetres. In decimal form we can write it as 4.3 centimetres.

Using numbers alone (without any unit of measure); we say

$$\frac{43}{10} = 4.3$$

Suppose we slightly change the question.

A wire of length 439 metres is cut into 100 equal pieces. What is the length of each piece ?

As before,

$$\frac{439}{100} \text{ metres} = 4\frac{39}{100} \text{ metres} = 4.39 \text{ metres}$$

In terms of numbers alone,

$$\frac{439}{100} = 4.39$$

Now a problem with numbers alone:

What is the decimal form of $\frac{4391}{1000}$?

$$\frac{4391}{1000} = 4\frac{391}{1000} = 4.391$$

Now, can't you write the decimal forms of these fractions ?

$$\frac{325}{10} =$$

$$\frac{325}{100} =$$

$$\frac{325}{1000} =$$

Did you notice the change in the position of decimal point ? Why is it so ?

We can split 325 as :

$$325 = (3 \times 100) + (2 \times 10) + 5$$

Taking $\frac{1}{10}$ of all,

$$\frac{325}{10} = (3 \times 10) + (2 \times 1) + \left(5 \times \frac{1}{10}\right)$$

That is to say, the positional values are all diminished to a tenth. The places shift one place to the right :

100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	
3	2	5				$\times \frac{1}{10}$
	3	2	5			

$$\frac{325}{10} = 32.5$$

This happens each time we take $\frac{1}{10}^{\text{th}}$.

100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	
3	2	5				$\times \frac{1}{10}$
	3	2	5			$\times \frac{1}{10}$
		3	2	5		$\times \frac{1}{10}$
		0	3	2	5	

That is,

$$\frac{325}{100} = \frac{325}{10} \times \frac{1}{10} = 32.5 \times \frac{1}{10} = 3.25$$

$$\frac{325}{1000} = \frac{325}{100} \times \frac{1}{10} = 3.25 \times \frac{1}{10} = 0.325$$

Can you complete this table, as above ?

100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$	
4	7	6					$\times \frac{1}{10}$
							$\times \frac{1}{10}$
							$\times \frac{1}{10}$
							$\times \frac{1}{10}$

Express these in decimal forms also

$$\frac{476}{10} =$$

$$\frac{476}{100} =$$

$$\frac{476}{1000} =$$

$$\frac{476}{10000} =$$

Now a question in the other direction:

What is the fractional form of 327.45 ?

What fraction, among $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \ldots$, of 32745 gives 327.45 ?

How many digits are there in the decimal part of 327.45 ?

So, what fraction should we take ?

$$327.45 = \frac{32745}{100}$$

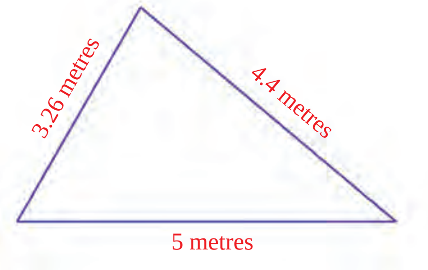
What about 327.045 ?

Can't you now write the following numbers as fractions ?

(i) 45.6 (ii) 45.06 (iii) 45.67 (iv) 4.506 (v) 456.07

Multiples

You have learnt in class 6, how to add and subtract measures in decimal form. Let's look at an example :

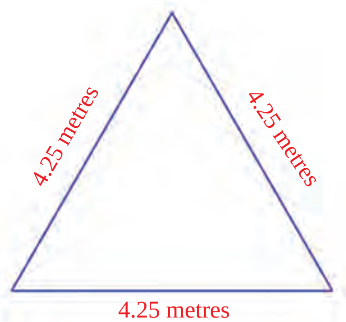


What is the perimeter of this triangle ?

$$5 + 4.4 + 3.26 = 5.00 + 4.40 + 3.26 = 12.66$$

Perimeter is 12.66 metres.

What about this triangle ?



Instead of writing

$$4.25 + 4.25 + 4.25 = 12.75$$

Let's write 3×4.25. How do we do this multiplication ?

4.25 metres means 425 centimetres, isn't it ?

So the perimeter in centimetres is $3 \times 425 = 1275$

Now we change this to metres. One centimetre is $\frac{1}{100}$ metre. So,

$$1275 \text{ centimetres} = \frac{1275}{100} \text{ metres} = 12.75 \text{ metres}$$

Instead of doing this in two steps like this, can we directly multiply 4.25 by 3 ?

For this, we first write 4.25 as a fraction.:

$$4.25 = \frac{425}{100}$$

Now let's multiply

$$3 \times 4.25 = 3 \times \frac{425}{100} = \frac{1275}{100} = 12.75$$

Can you do this problem similarly ?

A bottle can hold 1.25 litres of water. How many litres of water are needed to fill 4 such bottles ?

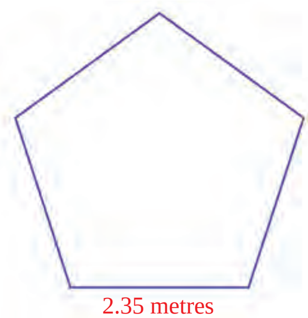
We need to find 4 times 1.25

$$4 \times 1.25 = 4 \times \frac{125}{100} = \frac{500}{100} = 5$$

So, we need 5 litres of water to fill the bottles.

Now try the following problems:

(1) The figure below shows a regular pentagon.



Find the perimeter of the pentagon.

(2) A kid needs 1.45 metres of cloth for a shirt. How many metres of cloth is needed for 4 shirts ?

(3) A bag holds 4.75 kilograms of rice. How much rice can 8 such bags hold ?

(4) A vessel full of oil was used to fill in 6 bottles. Each bottle holds 0.75 litre. How much oil was there in the vessel ?

Decimal multiplication

See this problem :

What is the area of a rectangle of length 8.5 centimetres and width 6.5 centimetres ?

We've seen that even when the lengths of the sides of a rectangle are fractions, its area is the product of the lengths of its sides. So, to find the area, we multiply 8.5 by 6.5, after converting them to fractions :

$$8.5 \times 6.5 = \frac{85}{10} \times \frac{65}{10} = \frac{85 \times 65}{10 \times 10} = \frac{5525}{100} = 55.25$$

That is, area = 55.25 square centimetres.

Let's try another problem :

It has been calculated that the weight of one millilitre of kerosene is 0.81 gram. What is the weight of 10.5 millilitres of kerosene ?

Let's do this without converting them to milligram and millilitre.

$$0.81 \times 10.5 = \frac{81}{100} \times \frac{105}{10} = \frac{81 \times 105}{1000} = \frac{8505}{1000} = 8.505$$

That is, the weight is 8.505 grams.

Now try these yourselves :

(1) Find the area in square metres of a rectangle of length 6.25 metres and width 4.2 metres.

(2) The weight of 1 millilitre of coconut oil is 0.91 gram. What is the weight of 10.5 millilitres of coconut oil ?

(3) The price of 1 litre of petrol is 110.12 rupees. What is the price of 2.5 litres of petrol ?

Multiplication operations

What is 314 × 12 ?

$$314 \times 12 = 314 \times (10 + 2) = (314 \times 10) + (314 \times 2) = 3140 + 628 = 3768$$

What is 31.4 × 12 ?

$$31.4 \times 12 = \frac{314}{10} \times 12 = \frac{314 \times 12}{10}$$

But we've already found 314 × 12

$$31.4 \times 12 = \frac{3768}{10} = 376.8$$

What about 314 × 1.2 ?

$$314 \times 1.2 = 314 \times \frac{12}{10} = \frac{314 \times 12}{10} = \frac{3768}{10} = 376.8$$

Likewise,

$$3.14 \times 12 = \frac{314}{100} \times 12 = \frac{314 \times 12}{100} = \frac{3768}{100} = 37.68$$

Also,

$$314 \times 0.12 = 314 \times \frac{12}{100} = \frac{314 \times 12}{100} = \frac{3768}{100} = 37.68$$

Can you quickly say what is 31.4 × 1.2 ?

How about 3.14 × 1.2 ?

Now let's look at them all together:

$$314 \times 12 = 3768$$
$$31.4 \times 12 = 376.8$$
$$314 \times 1.2 = 376.8$$
$$3.14 \times 12 = 37.68$$
$$314 \times 0.12 = 37.68$$
$$31.4 \times 1.2 = 37.68$$
$$3.14 \times 1.2 = 3.768$$

The digits in all the products are 3, 7, 6, 8 aren't they ?

The order of digits too remains the same.

What has changed ?

In the first product, there are no digits in the decimal places (that is in the places of $\frac{1}{10}, \frac{1}{100}, ...$).

In the second and third products, there is a digit in the $\frac{1}{10}$ th place. What about the next three ? And the last ?

What is the relation between the number of decimal places in the numbers to be multiplied and the number of digits in the decimal places of the product ?

Why is this so ?

The number of digits in the decimal places depends on the denominator of the fractional form, doesn't it ?

Look at 3.14 × 0.12

What is the denominator when 3.14 is written as fraction ?

What about the fractional form of 0.12 ?

What is the denominator of the product ?

What is the number of digits in the decimal part of the product, expressed as a decimal ?

Now try these problems :

(1) Given that $1234 \times 56 = 69104$

(i) Find the answers to the following problems without actual multiplication.

(i) 1.234×56

(ii) 12.34×5.6

(iii) 123.4×0.56

(iv) 1234 x 0.056

(ii) Like this, how many products can you find which gives 6.9104 ?

(2) In the following products, how many of them gives the same product as 1.234×5.67 ?

(i) 12.34×0.567

(ii) 1.234×567

(iii) 0.1234×5.67

(iv) 1.234×56.7

(v) 123.4×0.0567

(3) Find the greatest and least products from the following :

(i) 0.11×0.11

(ii) 1.1×1.1

(iii) 1.01×1.01

(iv) 0.101×1.1

(v) 10.1×0.101

Parts

A 10 metre long rope is cut into two equal pieces. How long is each piece ?

Half of 10 metres is 5 metres.

This can be written as a division :

$$10 \div 2 = 5$$

What if the rope is 10.4 metres ?

Half of 10 metres is 5 metres. What is half of 0.4 ? Is it 0.2 ?

To verify this, write 0.4 as a fraction :

$$\frac{1}{2} \times 0.4 = \frac{1}{2} \times \frac{4}{10} = \frac{4}{20} = \frac{2}{10} = 0.2$$

The length of each piece is 5.2 metres.

We can write it as a division

$$10.4 \div 2 = 5.2$$

Let's look at another problem :

A square was made with a cord of length 24.8 centimetres. What is the length of a side ?

One fourth of 24 is 6. What remains is 0.8 centimetre:

To find one-fourth of it, we write it as a fraction:

$$\frac{1}{4} \times 0.8 = \frac{1}{4} \times \frac{8}{10} = \frac{8}{40} = \frac{2}{10} = 0.2$$

So, the length of a side is 6.2 centimetres.

This also we can write as division

$$24.8 \div 4 = 6.2$$

Suppose we change the question slightly:

A square was made with a cord of length 23.2 centimetres. What is the length of a side ?

We can't do this one as we did before, can we ?

Let's write 23.2 centimetres itself as a fraction and find one fourth.

$$\frac{1}{4} \times 23.2 = \frac{1}{4} \times \frac{232}{10} = \frac{232}{40}$$

What next ?

The last fraction may be written as

$$\frac{232}{40} = \frac{58 \times 4}{10 \times 4} = \frac{58}{10} = 5.8$$

That is, the length of a side is 5.8 centimetres.

As a division,

$$23.2 \div 4 = 5.8$$

Another problem:

34.4 kilograms of rice was divided equally among 8 people. How many kilogram did each get ?

As in the earlier problem, we shall write 34.4 as a fraction and find the part required.

$$\frac{1}{8} \times 34.4 = \frac{1}{8} \times \frac{344}{10} = \frac{344}{80} = \frac{43 \times 8}{10 \times 8} = \frac{43}{10} = 4.3$$

That is, each gets 4.3 kilograms.

In other words,

$$34.4 \div 8 = 4.3$$

Now try these :

(1) The perimeter of an equilateral triangle is 12.9 centimetres. What is the length of each side ?

(2) 16.5 kilograms of rice was divided equally among 5 people. How many kilogram did each get ?

(3) A large vessel contains 25.2 litres of coconut oil. It was used to fill 6 small vessels of the same size. How much does each small vessel contain ?

(4) 33.6 kilograms of rice was divided equally among 8 people. Sujatha divided what she got into three equal parts and gave one part to Razia. How much did Razia get ?

(5) We have 7407 ÷ 6 = 1234.5. Use this result to find answers to the following questions without actual division :

(i) 740.7 ÷ 6   (ii) 74.07 ÷ 6   (iii) 7.407 ÷ 6

(iv) 0.7407 ÷ 6   (v) 0.07407 ÷ 6

Fraction and decimal

A 9 centimetres long wire was cut into two pieces of equal length. What is the length of each ?

$4\frac{1}{2}$ centimetres. How do we express it as a decimal ?

$\frac{1}{2}$ cetimetre is 5 millimetres. That is, $\frac{5}{10}$ centimetre. This we can write as 0.5 centimetre.

So, the length of a piece is 4.5 centimetres.

Let's look at the math part alone. What did we do to write $\frac{1}{2}$ as a decimal ?

From among several different fractional forms of $\frac{1}{2}$, we choose $\frac{5}{10}$.

$$\frac{1}{2} = \frac{5}{10} = 0.5$$

In the same way, can you find the decimal form of $\frac{1}{5}$?

Which fractional form of $\frac{1}{5}$ has 10 as the denominator ?

$$\frac{1}{5} = \frac{2}{10} = 0.2$$

Does $\frac{1}{4}$ have a decimal form ?

We get various forms of a fraction by multiplying the numerator and the denominator by the same number, right ?

No multiple of 4 is 10.

But $25 \times 4 = 100$. So,

$$\frac{1}{4} = \frac{25}{100} = 0.25$$

Like this,

$$\frac{3}{4} = \frac{25 \times 3}{25 \times 4} = \frac{75}{100} = 0.75$$

What about the decimal form of $\frac{1}{8}$?

10 and 100 are not multiples of 8.

Now,

$$8 = 2 \times 2 \times 2$$

2×5 gives 10; So multiplying 8 by $5 \times 5 \times 5$ will result in $10 \times 10 \times 10$, right ?

$(5 \times 5 \times 5) \times 8 = (5 \times 2) \times (5 \times 2) \times (5 \times 2) = 10 \times 10 \times 10.$

That is,

$$125 \times 8 = 1000$$

So,

$$\frac{1}{8} = \frac{125}{1000} = 0.125$$

In the same way

$$\frac{3}{8} = \frac{125 \times 3}{125 \times 8} = \frac{375}{1000} = 0.375$$

Also,

$$\frac{5}{8} = \frac{125 \times 5}{125 \times 8} = \frac{625}{1000} = 0.625$$

Now try these problems :

(1) Find the decimal forms of the following fractions.

(i) $\frac{3}{5}$ (ii) $\frac{4}{5}$ (iii) $\frac{1}{20}$ (iv) $\frac{7}{8}$

(2) 3 litres of milk is used to fill in 8 bottles of the same size. How many litres does each bottle hold ?

(3) A rope 17 metres long is cut into 25 equal pieces. What is the length of each piece in metres ?

(4) 19 kilograms of rice was equally divided among 20 people. How much kilograms did each get ?

Now try this problem :

8.5 kilograms of rice was equally packed into 2 bags. How much does each bag contain ?

As we did in the case of some earlier problems, we express 8.5 as a fraction and halve it.

$$\frac{1}{2} \times 8.5 = \frac{1}{2} \times \frac{85}{10} = \frac{85}{20} = \frac{17 \times 5}{4 \times 5} = \frac{17}{4} = 4\frac{1}{4} = 4.25$$

That is, each bag contains 4.25 kilograms.

Another problem :

10.5 litres of water was used to fill 6 bottles of the same size. How much litres does each bottle contain ?

We write 10.5 as a fraction and find $\frac{1}{6}$ of it.

$$\frac{1}{6} \times \frac{105}{10} = \frac{105}{60} = \frac{21 \times 5}{12 \times 5} = \frac{21}{12} = \frac{7 \times 3}{4 \times 3} = \frac{7}{4} = 1\frac{3}{4} = 1.75$$

One bottle contains 1.75 litres

Can't you now do the following problems ?

(1) A ribbon 14.5 centimetres long is cut into two equal pieces. What is the length of each piece in centimetres ?

(2) What is the length of a side of a square of perimeter 20.5 metres ?

(3) The price of 6 pens is 40.50 rupees. What is the price of one pen ?

Decimal division

Let's take another look at a problem in the lesson, **Reciprocals** :

$5\frac{1}{4}$ litres of water is to be used to fill $\frac{3}{4}$ litre bottles. How many bottles are needed ?

How do we do this ?

Thinking in terms of numbers alone, the question is "which number multiplied by $\frac{3}{4}$ gives $5\frac{1}{4}$?"

So the question becomes :

$5\frac{1}{4}$ is $\frac{3}{4}$ of which number ?

To calculate this, we saw in the lesson Reciprocals, that it was enough to find $\frac{4}{3}$ times $5\frac{1}{4}$

$$\frac{4}{3} \times 5\frac{1}{4} = \frac{4}{3} \times \frac{21}{4} = \frac{21}{3} = 7$$

That is, 7 bottles are needed.

Suppose the measures in this question were in decimals.

5.25 litres of water is to be used to fill in 0.75 litre bottles. How many bottles are needed ?

We convert all measures into fractions.

$$5.25 = \frac{525}{100}$$

$$0.75 = \frac{75}{100}$$

We can simplify the fractions to $\frac{21}{4}$ and $\frac{3}{4}$ and continue as earlier.

Or we can find $\frac{100}{75}$ times $\frac{525}{100}$.

$$\frac{100}{75} \times \frac{525}{100} = \frac{525}{75} = \frac{7 \times 3 \times 25}{3 \times 25} = 7$$

As a division,

$$5.25 \div 0.75 = 7$$

Now try this problem :

The area of a rectangle is 3.25 square metres and the length is 2.5 metres. What is its width ?

Keeping aside the measures, the question is "which number multiplied by 2.5 gives 3.25 ?"

Thus the question is

3.25 is 2.5 times of what number ?

To find this, we convert the numbers into fractions :

$$3.25 = \frac{325}{100}$$

$$2.5 = \frac{25}{10}$$

What we have to find is, $\frac{325}{100}$ is $\frac{25}{10}$ times of what number ?

In order to find it, we can use reciprocals.

$$\frac{10}{25} \times \frac{325}{100} = \frac{325}{25 \times 10} = \frac{13}{10} = 1.3$$

So, the width is 1.3 metres.

As a division,

$$3.25 \div 2.5 = 1.3$$

Now try these problems:

(1) A vessel contains 4.05 litres of coconut oil. It is to be used to fill 0.45 litre bottles. How many bottles are needed ?

(2) An iron rod 17.5 metres long is cut into pieces of length 2.5 metres each. How many pieces are there ?

(3) 6.5 kilograms of chilli powder was packed in 0.25 kilogram packets. How many packets are there ?

6 RATIO

Rectangle problem

Look at these pictures:

The same picture, but different sizes.

Now look at this picture :

Is it like the first picture ?

The first three pictures are not squares, the height is less than the width.

What about the fourth picture ?

The width is the same as that of the third picture, but the height is more (and the shape of the elephant is also not correct). So, when the size of a picture is changed, the width and height should change in the same way.

Let's be more clear. In the first three pictures, the height of the rectangle is $\frac{3}{4}$ of the width. In the fourth, the height is equal to the width.

In mathematics, we say it this way

In the first three rectangles, the height and width are in the ratio 3 to 4.

3 to 4 is written as 3 : 4. Then we can slightly shorten the above statement:

In the first three rectangles, the height and width are in the ratio 3 : 4

In several smartphone cameras, the height to width ratio can be set in three ways.

First, the ratio 3 : 4, we saw just now.

Second, 9 : 16. What does it mean ?

The height is $\frac{9}{16}$ of the width.

The third is 1 : 1.

The height and width are equal; that is, a square picture.

Look at three pictures of the same scene in these three different ratios.

3 : 4

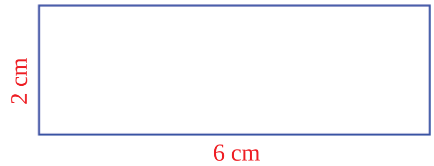
9 : 16

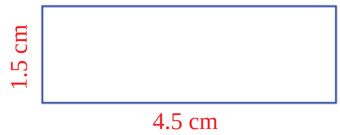
1 : 1

(The square picture of the elephant we saw earlier, was not actually shot in this ratio. It was artificially transformed using computer tools and so the elephant got distorted). Here 3 : 4 and 9 : 16 give the ratio of the height to the width. If we want to state the ratio of the width to the height we must say 4 : 3 and 16 : 9.

If the height is $\frac{3}{4}$ of the width, then the width is $\frac{4}{3} = 1\frac{1}{3}$ times the height, right ? (See the lesson **Reciprocals**)

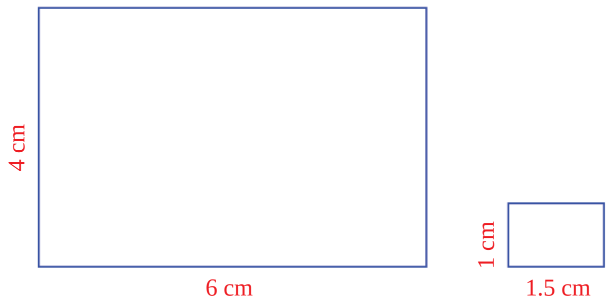
Let's look at some more rectangles:





In each of these, the width is three times the height. So the ratio of the height to the width is 1 : 3.

Now look at these rectangles.

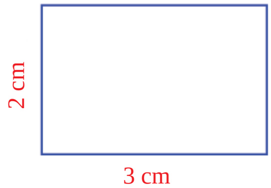


In each, the width is one and a half times the height, isn't it ?

How do we state it as a ratio ?

We can say one and a half to one. But we usually avoid fractions while stating ratios.

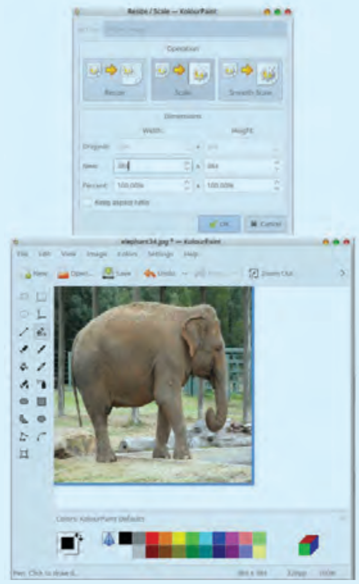
Suppose we take the height to be 2 centimetres ? What is $1\frac{1}{2}$ times 2 ?



Picture ratio

We can not only draw pictures, but change their sizes also using **Kolour Paint** in KITE GNU-Linux. For this we click **Image** → **Resize/Scale** :

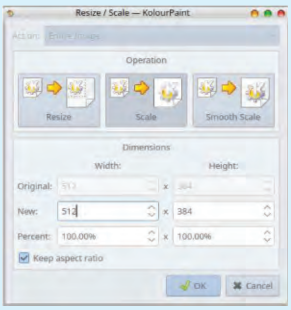
In the pop-up window we can change width and height.



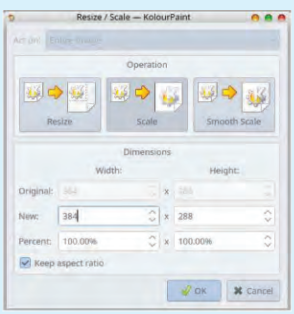
Aspect ratio

The height to width ratio of a rectangular picture is called its aspect ratio.

To keep the shape of the picture the same, when we enlarge or diminish the picture, this ratio should be kept the same. In the window which helps change the size in **Kolour Paint**, if we check the **Keep aspect ratio** box, we can change only one of height or width.



To keep the aspect ratio the same, the other length will be calculated and it will change accordingly.



The picture will be enlarged or shrunk without changing the shape.

So we say that in these rectangles, the height and width are in the ratio 2 to 3 and write 2 : 3.

Can't we state this ratio as 4 : 6 ? There's nothing wrong in it. But we usually express ratios using the smallest numbers possible.

How do we state the following using ratio ?

The width of a rectangle is two and a half times the height.

If the height is 1 centimetre, the width would be $2\frac{1}{2}$ cm. What if the height is taken as 2 centimetres ?

Now we have the answer.

The height and length of the rectangle are in the ratio 2 : 5.

What if the width is one and a quarter times the height ?

If the height is taken as 2 centimetres, then the width would be $2\frac{1}{2}$ centimetres. We haven't got rid of the fraction. What if the height is taken as 4 centimetres ?

So, in such rectangles, the height and width are in the ratio 4 : 5.

Did you notice another thing in these cases ?

If we extend the height and width by the same multiple or shorten them by the same fraction, the ratio remains the same. For example, look at the heights and widths given below.

Height	**Width**
3 centimetres	4 centimetres
6 metres	8 metres
1 metre	$1\frac{1}{3}$ metres
$\frac{3}{4}$ metre	1 metres

In all these, the width is $\frac{4}{3} = 1\frac{1}{3}$ times the height. In other words, the height is $\frac{3}{4}$ of the width.

In terms of ratios,

The height and width are in the ratio 3 : 4;

The width and height are in the ratio 4 : 3

Now try these problems :

(1) Write down the ratio of the height to width of each of the following rectangles using the smallest possible natural numbers.

(i) Height 8 centimetres, width 10 centimetres

(ii) Height 8 metres; width12 metres

(iii) Height 20 centimetres, width 1 metre

(iv) Height 40 centimetres; width 1 metre

(v) Height 1.5 centimetres; width 2 centimetres

(2) In the table below, the height, width and their ratio of some rectangles are given, but only two of each. Can you calculate the third and complete the table.

Height (cm)	Width (cm)	Ratio
6	8	
3		3 : 4
$\frac{3}{4}$	1	
	1	3 : 4
6	15	
2		2 : 5
1		2 : 5
	1	2 : 5

Flags

Suppose we draw the picture of our National Flag. It isn't enough that we get the colours right. The ratio of the height to width must also be correct. It is 2 : 3. That is, when we draw our national flag, if the width is 3 centimetres, the height has to be 2 centimetres.

This ratio is different for various countries. For example, in the case of Australia, the ratio 1 : 2

In the German flag the ratio is 3 : 5

Other measures

We can use ratio to express any two lengths as multiple and part other than just the height and width of a rectangle.

For example, consider two pieces of ropes 4 metres, and 6 metres long. $\frac{1}{6}$ of 6 metres is 1 metre and 4 times of this is 4 metres. So, $\frac{4}{6}$ of 6 metres is 4 metres. (See the lesson **Reciprocals**).

Since $\frac{4}{6}=\frac{2}{3}$, we can say that the smaller and larger pieces are in the ratio 2 : 3. Thus when we say that two lengths are in the ratio 2 : 3, it means that the length of the shorter one is $\frac{2}{3}$ the length of the longer. We don't know what the actual lengths are. They could be any of these :

2 metres,	3 metres
4 metres,	6 metres
6 metres,	9 metres
1 metre,	$1\frac{1}{2}$ metres
3 metres,	$4\frac{1}{2}$ metres
$\frac{1}{2}$ metre,	$\frac{3}{4}$ metre

These could be in centimetre or kilometre instead of metre. One thing is that the shorter length is $\frac{2}{3}$ of the longer.

We can use ratios to express any two measures (not just lengths) as multiples and parts.

For example, if we have a 15 litre bucket and a 25 litre bucket, then the smaller bucket holds $\frac{15}{25}$ of what the larger holds, isn't it ?

Since $\frac{15}{25}=\frac{3}{5}$, we can say that the ratio of the capacity of the smaller to the larger buckets is 3 : 5.

We can say counts also in terms of ratio. For example, if there are 12 boys and 21 girls in a class, the number of boys is $\frac{12}{21}$ of the number of girls, isn't it ?

$\frac{12}{21} = \frac{4}{7}$ and so the number of boys and girls are in the ratio 4 : 7.

Calculate the ratio, using the smallest possible numbers, in each of the following problems:

(1) Amina has 105 rupees with her and Mercy has 175 rupees. What is the ratio of the smaller amount to the larger ?

(2) 96 women and 144 men attended a meeting. Find the ratio of the number of women to the number of men.

(3) Of two pencils, the shorter is 4.5 centimetres long and the longer 7.5 centimetres. What is the ratio of the length of the longer to the shorter ?

(4) When a rope was used to measure the sides of a rectangle, the width was $\frac{1}{4}$ of the rope and the height was $\frac{1}{3}$ of the rope. What is the ratio of the height to the width ?

Motion and Ratio

Have you taken apart toy carts or old clocks ? And seen toothed wheels of various sizes in them ? See this picture.

This is a small part of a machine. The smaller wheel has 13 teeth and the larger 21. When the small wheel rotates 21 times, the large one would have rotated only 13 times. The speed of rotation is controlled in machines by setting the ratio of the number of teeth in the wheels suitably.

(5) $3\frac{1}{2}$ glasses of water would fill a large bottle and $2\frac{1}{2}$ glasses of water could fill a smaller bottle. What is the ratio of the capacities of the larger bottle to the smaller bottle ?

Ratio of mixtures

Ammu's mother usually grinds two cups of rice and one cup of black gram to make idlis. On the day before some guests would be visiting, she took four cups of rice. How many cups of black gram should she take ?

To get the same consistency and flavour every time, the amount of black gram should be half that of rice. So, for four cups of rice, she should take two cups of black gram.

We may say that rice and black gram should be in the ratio 2 : 1

Another problem of mixing:

The walls of Aji's house needs painting. First 25 litres of green paint and 20 litres of white paint were mixed. This wasn't enough. Another 15 litres of green was then taken. How many litres of white paint should be mixed with this ?

To get the same shade of green, the ratio of white and green is to be the same.

In what ratio was green and white mixed first ?

5 litres of green for every 4 litres of white.

To keep this ratio, how much white paint should be mixed with 15 litres of green ?
How many times 5 is 15 ?

So 3 times 4 litres of white should be mixed with the green. That means 12 litres.

To get the same shade of green, how many litres of green should be mixed with 16 litres of white?

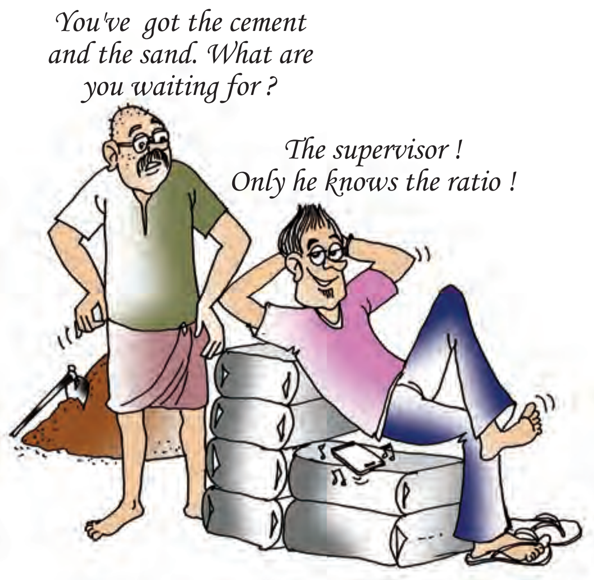
Now try the following problems :

(1) To make dosa, we need to take 2 cups of black gram for every 6 cups of rice. For 9 cups of rice, how many cups of black gram shall be taken ?

(2) To plaster the walls of a house, cement and sand are mixed in the ratio 1 : 5. 45 sacks of cement were bought. How many sacks of sand are needed ?

(3) 12 litres of paint was mixed with 8 litres of turpentine while painting the house. How many litres of turpentine should be mixed with 15 litres of paint ?

(4) In a ward of a panchayat, the women and men are in the ratio 11 : 10. There are 1793 women in the ward. How many men are there in the ward ? What is the total number of women and men ?



Division problems

We've seen that rice and black gram are to be mixed in the ratio 2 : 1 to make idlis. In 9 cups of such a mixture, how much is the rice?

2 cups of rice and 1 cup of black gram makes 3 cups.

How many times 3 is 9 ?

To keep the ratio, rice and black gram should be made threefold. That gives 6 cups of rice and 3 cups of black gram.

Let's look at another problem :

600 men and 400 women are members of a co-operative society. A working committee of 30 members is to be formed. The number of men and women in the committee should be in the same ratio as the number of men and women in the society. How many men and how many women should be there in the committee ?

We know that the number of men and women in the society are in the ratio 3 : 2.

3 men and 2 women make 5 members.

Here we need 30 members.

How many times 5 is 30 ?

So there should be 3 × 6 = 18 men and 2 × 6 = 12 women in the committee

One more problem:

The school needs a vegetable garden. A rectangular plot is to be roped off for this. The length of the rope is 32 metres. They decided to have width and length in the ratio 3 : 5. What should be the width and length ? The length of the rope is 32 metres. So, the perimeter of the rectangle is also 32 metres.

If we take the width and length as 3 metres and 5 metres, what is the perimeter ?

How many times of 16 is 32 ?

$$\frac{32}{16} = 2$$

So width is 2 times 3 metres, that is 6 metres. Length is 2 times 5 metres, that is 10 metres.

Now try these problems :

(1) Suhara and Sita started a business. Suhara invested 40000 rupees and Sita 50000 rupees. They made a profit of 9000 rupees. It was divided in the ratio of their investments. How much did each get ?

(2) Ramesan and John took up a contract for a work. Ramesan worked the first 6 days and John 7 days. They got 6500 rupees. They divided it in the ratio of the number of days each worked. How much did each get ?

(3) When Ramu and Raju divided a sum of money in the ratio 3 : 2, Ramu got 480 rupees.

(i) How much did Raju get ?

(ii) What was the sum that was divided ?

(4) Draw a line *AB*, 9 centimetres long. Mark a point *P* on it. The lengths of *AP* and *PB* should be in the ratio 1 : 2. How far away from *A* should we mark *P* ? Calculate and mark it.

(5) Draw a line 15 centimetres long. Mark a point on it that divides the line in the ratio 2 : 3. Calculate the length and mark the point.

(6) Draw a rectangle of perimeter 30 centimetres and sides of length in the ratio 1 : 2.

(i) With the same perimeter draw two more rectangles with sides in the ratio 2 : 3 and 3 : 7.

(ii) Calculate the areas of the three rectangles. Which rectangle has the greatest area?

7 Shorthand Math

Numbers and letters

What is the perimeter of a square of side 5 metres ?

Perimeter is the sum of the lengths of all sides. So, in this problem we need to add four 5's, That is, 4 times 5.

$$\text{Perimeter} = 4 \times 5 \text{ metres} = 20 \text{ metres}$$

If the length of the sides is something other than 5 metres, then again, the method of calculating the perimeter remains the same, doesn't it ?

We may generalize this :

The perimeter of a square is four times the length of a side.

Shortening a bit,

$$\text{Perimeter of a square} = 4 \times \text{Length of a side}$$

For further shortening, we can denote the length of a side by the letter s and the perimeter by the letter p.

Taking the side of a square as s and the perimeter as p

$$p = 4 \times s$$

This method of denoting relations between measures using letters was seen in class 6 in the lesson **Letter Math**.

We also saw some features of this:

- Write products without multiplication sign
- In products with number and letter, write the number first.

So, we can write the relation between the length of a side of a square and its perimeter like this:

If the length of the side of a square is s and the perimeter is p, then

$$p = 4s$$

The relation between mere numbers can also be written in shorthand using letters.

For example, consider this

A number added to itself is twice the number.

In the language of math, we write :

a number + the same number = 2 × number

Let's denote the number by n. Then

$$n + n = 2n, \text{ for any number } n$$

We may use any letter instead of n to denote the number.

$$x + x = 2x, \text{ for any number } x$$

Continuing, the fact that a number added to twice the number gives three times the number can be written

$$2x + x = 3x, \text{ for any number } x$$

Like this, twice a number added to thrice the number gives five times the number can be written

$$3x + 2x = 5x, \text{ for any number } x$$

Any number multiplied by 1 gives the same number, can be written as

$$x \times 1 = x, \text{ for any number } x$$

Again, the fact that any number divided by 1 gives the same number, can be written as

$$x \div 1 = x, \text{ for any number } x$$

Usually, we write division as a fraction in this shorthand using letters. So the above statement is written as :

$$\frac{x}{1} = x, \text{ for any number } x$$

We need to use more than one letter when we talk about several numbers.

For example, adding 3 to 5 and then subtracting 5 from the sum gives 3, doesn't it ?

$$(5 + 3) - 5 = 3$$

Any numbers in place of 5 and 3 would give the same result. That is,

If we add a number to another number and then subtract the original number, we get the added number.

How do we write this using our shorthand?

First we express each step using letters.

(i) Let x be the first number and y be the number added

(ii) y added to x is $x + y$

(iii) The first number subtracted from the sum is $(x + y) - x$

(iv) It is actually the number added $(x + y) - x = y$

This can be written:

$$(x + y) - x = y, \text{ for any numbers } x, y$$

This method of stating relations between measures or numbers using letters is called *algebra*.

What we have seen here is the method of stating in algebra, certain things stated in ordinary language. For example,

Ordinary language	Algebraic language
Twice a number added to thrice the number gives five times the number.	$3x + 2x = 5x$, for any number x
A number added to a number and then the original number subtracted gives the number added.	$(x + y) - x = y$, for any numbers x, y

Now can't you write the following statements using the language of algebra ?

(1) Zero added to any number gives the same number.

(2) Zero subtracted from any number gives the same number.

(3) Any number subtracted from the same number gives zero.

(4) Any number multiplied by zero gives zero.

(5) Any number divided by the same number gives 1.

(6) Twice a number added to the number makes three times the number.

(7) Twice a number subtracted from thrice the number gives the number.

(8) A number added to a number, and then the added number subtracted gives the original number.

One by one and altogether

Another problem :

28 is added to 15, then 2 also is added. What is the result ?

Can't you find the answer ? How did you get it ?

Did you do it this way ?

$$15 + 28 = 43$$
$$43 + 2 = 45$$

Instead of first adding 28 to 15 and then adding 2, we can easily add 30 directly to 15.

Some times, instead of adding two numbers one after the other to another number, it is easier to add the sum of these numbers.

This can be done for any three numbers, right ?

That is,

Adding two numbers one after another to a number, or adding their sum, gives the same result.

How do we write this is in the language of algebra ?

(i) Let x be the first number and let y, z be the numbers to be added.

(ii) First adding y to x, and then adding z is $(x + y) + z$

(iii) The sum of the two numbers that are added is $y + z$; this added to x is $x + (y + z)$

(iv) Both give the same answer $(x + y) + z = x + (y + z)$

Now we can state the whole thing in the language of algebra :

$(x + y) + z = x + (y + z)$, for any three numbers x, y, z

There are also situations where adding one after another is easier than adding the sum.

For example, suppose we want to calculate 25 + 18. If we recall 18 = 5 + 13, then we can first add 5 to get 30 and then add 13 to get 43, all in our heads.

Now, another problem :

Subtract 8 from 25 and then subtract 2 from it.

How did you do it ? Like this ?

$$25 - 8 = 17$$

$$17 - 2 = 15$$

Or did you subtract 10 to get $25 - 10 = 15$ immediately ?

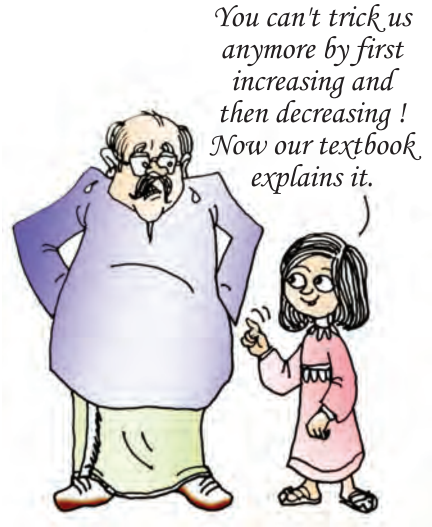
Subtracting 8 from 25 means taking away 8 from 25

What if we take away 2 more ?

Means, 10 taken away in all, right ?

Try other numbers and think about this.

How can you state it as a general principle ?



Subtracting two numbers one after the other from a number or subtracting the sum of these two numbers from the first number, both give the same result.

In the language of algebra.

$(x - y) - z = x - (y + z)$, for any three numbers x, y, z

As in the case of addition, it might be sometimes easier to subtract the numbers one after the other, rather than subtracting the sum.

For example, to subtract 201 from 500, we first subtract 200 to get 300; and then subtract 1 to get 299. Isn't it easier ?

Now do the following problems mentally:

(1) (i) $49 + 125 + 75$

(ii) $3\frac{1}{2} + 8\frac{3}{4} + \frac{1}{4}$

(iii) $15.5 + 0.25 + 0.75$

(iv) $38 + 27$

(v) $136 + 64$

(2) (i) $(135 - 73) - 27$

(ii) $\left(37 - 1\frac{1}{2}\right) - \frac{1}{2}$

(iii) $(298 - 4.5) - 3.5$

(iv) $78 - 29$

(v) $140 - 51$

Need for brackets

Adding 7 and 4 and then adding 2, or adding 4 and 2 first and then adding this to 7, we get 13 either way. That is,

$(7 + 4) + 2 = 7 + (4 + 2)$

So we may write this sum as $7 + 4 + 2$ without brackets. But if we subtract 4 from 7 and then subtract 2 we get 1, whereas if we subtract 2 from 4 and then subtract it from 7 we get 5. That is,

$$(7 - 4) - 2 = 1$$

$$7 - (4 - 2) = 5$$

So, if we just write $7 - 4 - 2$, the answer will be different depending on which operation we do first. So, we must use brackets to clearly show the order of operations.

Addition and subtraction

Look at this problem:

There were 38 children at the beginning of the class. 5 children came late. After sometime 3 went to attend a meeting of the Math Club. How many children are there in the class now ?

Let's calculate it in the order of events :

(i) When 5 more children came $38 + 5 = 43$

(ii) When 3 children left $43 - 3 = 40$

Thinking about all that happened at one go, we can also calculate like this :

(i) 5 students came and 3 left. So only $5 - 3 = 2$ more

(ii) The 38 children originally in the class, together with these 2, makes $38 + 2 = 40$

What did we do in this problem ?

Instead of adding a number and subtracting another, we added what was got by subtracting the second number from the first.

Look at another example :

$$(108 + 25) - 15 = 108 + (25 - 15) = 108 + 10 = 118$$

We must be careful about something here. To calculate in this manner, the number added has to be greater than the number subtracted. For example, look at this problem:

$$(25 + 10) - 15$$

To calculate it this way, we must subtract 15 from 10. But this is not possible. So how do we state this as a general principle ?

Starting with a number, if we add a larger number and then subtract a smaller number, or add the difference of the smaller number from the larger, either way we get the same result.

What is the algebraic form of this ?

$(x + y) - z = x + (y - z)$, for any three numbers x, y, z with $y > z$.

Some times it is more convenient to apply it in the reverse. For example, look at this problem.

What is 99 added to 25 ?

Think of 99 as 1 less than 100. Then, instead of adding 99, we add 100 first and subtract 1.

This we can do mentally as,100 added to 25 gives 125, subtracting 1 gives 124.

Let's write out what we did :

$$25 + (100 - 1) = (25 + 100) - 1$$

This is an application of the algebraic statement given above read in reverse :

$$x + (y - z) = (x + y) - z$$

Now you can try to calculate these mentally :

(i) $(136 + 29) - 19$

(ii) $\left(3\frac{1}{2} + 5\frac{3}{4}\right) - 2\frac{1}{4}$

(iii) $(298 + 14.5) - 12.5$

(iv) $23 + (35 - 18)$

(v) $65 + 98$

We have seen addition followed by subtraction. Now let's look at subtraction followed by addition.

Gopu has 110 rupees in his cash box. He took out 15 rupees to buy a pen. He bought a pen for 10 rupees and put the remaining 5 rupees in the box. How much money is there in the box now ?

We first do the problem in the order in which Gopu did things:

(i) When he took out 15 rupees, money in the box was 110 – 15 = 95 rupees

(ii) When he put back 5 rupees, it became 95 + 5 = 100 rupees

We can also think back after everything he did, like this:

(i) He took 15 rupees and put back 5 rupees. So money in the box is reduced by 15 – 5 = 10 rupees.

(ii) Amount in the box now is 110 – 10 = 100 rupees

How do we calculate (234 – 45) + 15 like this ?

Taking away 45 and then putting in 15 means only 45 – 15 = 30 is taken away in effect. Thinking like this we can quickly get the answer as 234 – 30 = 204.

Can we do like this, all problems of first subtracting and then adding ?

For example, we can't do (29 – 7) + 17 like this.

So we state this method as a general principle like this :

Starting with a number, if we subtract a larger number and then add a smaller number, or subtract the difference of the smaller from the larger, either way we get the same result.

And thus in the language of algebra

$$(x - y) + z = x - (y - z), \text{ for any } x, y, z \text{ with } y > z$$

Sometimes, it is useful to read this in reverse :

$$x - (y - z) = (x - y) + z$$

For example, suppose we have to subtract 99 from 237. If we think of 99 as 1 less than 100, then we can see that instead of subtracting 99, we can subtract 100 and then add 1. Now we can easily subtract 100 from 237 to get 137 and then add 1 to get 138.

That is,

$$237 - 99 = 237 - (100 - 1) = (237 - 100) + 1 = 137 + 1 = 138$$

Try to do these problems in head :

(i) (135 – 73) + 23

(ii) $(38 - 8\frac{1}{2}) + \frac{1}{2}$

(iii) (19 – 6.5) + 2.5

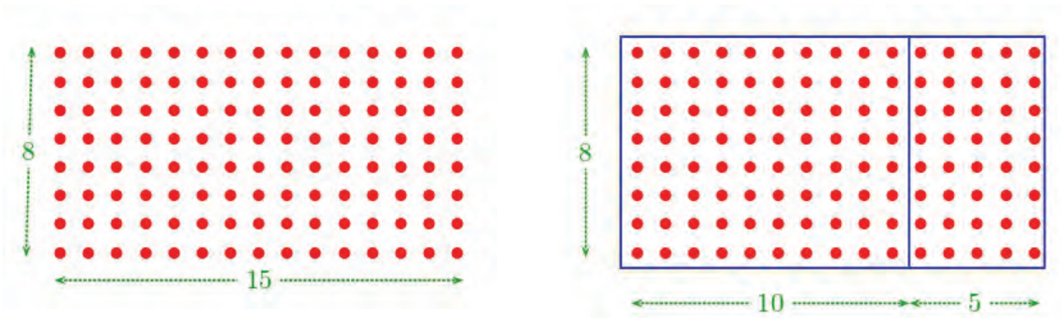
(iv) 135 – (35 – 18)

(v) 240 – (40 – 13)

Addition and subtraction and then multiplication

How do we calculate 15 × 8 ?

$$15 \times 8 = (10 + 5) \times 8 = (10 \times 8) + (5 \times 8) = 80 + 40 = 120$$



Whether we multiply 8 by 15, or multiply by 10 and 5 separately and add, we get 120. This is true for all numbers, isn't it ?

For example, how do we calculate $3\frac{1}{2} \times 4$?

$$3\frac{1}{2} \times 4 = \left(3 + \frac{1}{2}\right) \times 4 = (3 \times 4) + \left(\frac{1}{2} \times 4\right) = 12 + 2 = 14$$

This may be stated as a general principle :

Multiplying a sum by a number, or multiplying each number in the sum separately and adding, either way gives the same result.

In the language of algebra :

$(x + y)z = xz + yz$, for any three numbers x, y, z

We can do multiplication by a difference, instead of sum in the same way. For example, suppose we want to find 19 × 25. We can use 19 = 20 – 1 and write

$$19 \times 25 = (20 - 1) \times 25 = (20 \times 25) - (1 \times 25) = 500 - 25 = 475$$

This can also stated as a general principle :

Multiplying a difference by a number, or multiplying each number in the difference and subtracting, either way gives the same result.

In the language of algebra :

$(x - y)z = xz - yz$, for any three numbers x, y, z

Reading these in reverse is also useful in some problems.

$$xz + yz = (x + y)z$$
$$xz - yz = (x - y)z$$

For example, we can calculate the sum of half of 35 and half of 15 like this :

$$\left(\frac{1}{2} \times 35\right) + \left(\frac{1}{2} \times 15\right) = \frac{1}{2} \times (35 + 15) = \frac{1}{2} \times 50 = 25$$

And their difference like this :

$$\left(\frac{1}{2} \times 35\right) - \left(\frac{1}{2} \times 15\right) = \frac{1}{2} \times (35 - 15) = \frac{1}{2} \times 20 = 10$$

Now try to do these mentally:

(i) 103×15

(ii) 98×25

(iii) $(63 \times 12) + (37 \times 12)$

(iv) $(65 \times 11) - (55 \times 11)$

(v) $\left(15 \times \frac{3}{4}\right) + \left(5 \times \frac{3}{4}\right)$

(vi) $\left(5\frac{1}{2} \times 23\right) - \left(4\frac{1}{2} \times 23\right)$

CONSTITUTION OF INDIA

Part IV A

FUNDAMENTAL DUTIES OF CITIZENS

ARTICLE 51 A

Fundamental Duties- It shall be the duty of every citizen of India:

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers, wild life and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievements;

(k) who is a parent or guardian to provide opportunities for education to his child or, as the case may be, ward between age of six and fourteen years.

CHILDREN'S RIGHTS

Dear Children,

Wouldn't you like to know about your rights? Awareness about your rights will inspire and motivate you to ensure your protection and participation, thereby making social justice a reality. You may know that a commission for child rights is functioning in our state called the **Kerala State Commission for Protection of Child Rights.**

Let's see what your rights are:

- Right to freedom of speech and expression.
- Right to life and liberty.
- Right to maximum survival and development.
- Right to be respected and accepted regardless of caste, creed and colour.
- Right to protection and care against physical, mental and sexual abuse.
- Right to participation.
- Protection from child labour and hazardous work.
- Protection against child marriage.
- Right to know one's culture and live accordingly.
- Protection against neglect.
- Right to free and compulsory education.
- Right to learn, rest and leisure.
- Right to parental and societal care, and protection.

Major Responsibilities

- Protect school and public facilities.
- Observe punctuality in learning and activities of the school.
- Accept and respect school authorities, teachers, parents and fellow students.
- Readiness to accept and respect others regardless of caste, creed or colour.

Contact Address:

Kerala State Commission for Protection of Child Rights

'Sree Ganesh', T. C. 14/2036, Vanross Junction

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Email: childrights.cpcr@kerala.gov.in, rte.cpcr@kerala.gov.in

Website : www.kescpcr.kerala.gov.in

Child Helpline - 1098, Crime Stopper - 1090, Nirbhaya - 1800 425 1400

Kerala Police Helpline - 0471 - 3243000/44000/45000

Online ***R. T. E Monitoring*** *: www.nireekshana.org.in*