Standard VIII

MATHEMATICS

Part - 2

Government of Kerala
Department of Education
2016

State Council of Educational Research and Training (SCERT)

KT-511-1/Maths-8(E) Vol-2

THE NATIONAL ANTHEM

Jana-gana-mana adhinayaka, jaya he
Bharatha-bhagya-vidhata.
Punjab-Sindh-Gujarat-Maratha
Dravida-Utkala-Banga
Vindhya-Himachala-Yamuna-Ganga
Uchchala-Jaladhi-taranga
Tava subha name jage,
Tava subha asisa mage,
Gahe tava jaya gatha.
Jana-gana-mangala-dayaka jaya he
Bharatha-bhagya-vidhata.
Jaya he, jaya he, jaya he,
Jaya jaya jaya, jaya he!

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give respect to my parents, teachers and all elders and treat everyone with courtesy.

I pledge my devotion to my country and my people. In their well-being and prosperity alone lies my happiness.

Prepared by:

State Council of Educational Research and Training (SCERT)
Poojappura, Thiruvananthapuram 695 012, Kerala

Website: www.scertkerala.gov.in
E-mail: scertkerala@gmail.com
Phone: 0471-2341883, Fax: 0471-2341869
Typesetting and Layout: SCERT
First Edition : 2015, Reprint : 2016
Printed at: KBPS, Kakkanad, Kochi-30

Dear children,

We have travelled much
In the world of math
Let's continue
Our explorations
Our findings
We have much more to go ahead
In Mathematics
To the ever widening world of Numbers
To the precise reasoning of Geometry
To the higher levels to Algebra
Let's continue our search...

Dr. P. A. Fathima
Director
SCERT

TEXTBOOK DEVELOPMENT

Participants

T. P. Prakashan
GHSS, Vazhakadu, Malappuram

Unnikrishnan M. V.
GHSS, Kumbala, Kasaragode

Narayanan K.
BARHSS, Bovikkanam,Kasaragode

Mohanan C.
GHSS, Angadikkal South, Chenganoor

Ubaidulla K. C.
SOHSS, Areacode, Malappuram

Vijayakumar T. K.
GHSS, Cherkkala, Kasaragode

Sreekumar
GGHSS, Karamana, Thiruvananthapuram

V. K. Balagangadharan
GMHSS, Calicut University Campus
Malappuram

Narayananunni
DIET, Palakkad

Abraham Kurian
CHSS, Pothukallu, Nilambur

Sunil Kumar V. P.
Janatha HSS, Venjaramoodu
Thiruvananthapuram

Krishnaprasad M.
PMSA HSS, Chappangadi
Malappuram

Cover

Rakesh P. Nair

Experts

Dr. E. Krishnan
Prof. (Rtd) University College, Thiruvananthapuram

Academic Co-ordinator
Sujith Kumar G.
Research Officer, SCERT

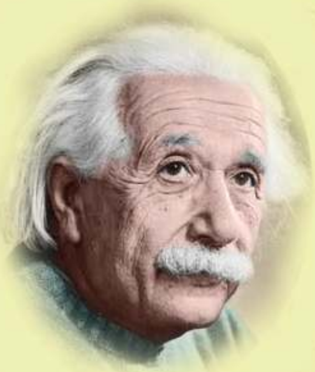
ENGLISH VERSION

Dr. E. Krishnan
Prof.(Rtd) University College,
Thiruvananthapuram

Venugopal C.
Asst. Professor,
Govt. College of Teacher Education,
Thiruvananthapuram

Academic Co-ordinator
Sujith Kumar G.
Research Officer, SCERT

State Council of Educational Research and Training (SCERT)
Vidya Bhavan, Thiruvananthapuram



Contents

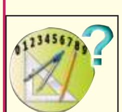
Certain icons are used in this textbook for convenience

Computer Work

Additional Problems

Project

Self Assessment



For Discussion

6

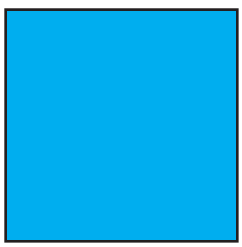
Construction of Quadrilaterals

Classification

We have seen different kinds of quadrilaterals. Let's look at their properties once more:

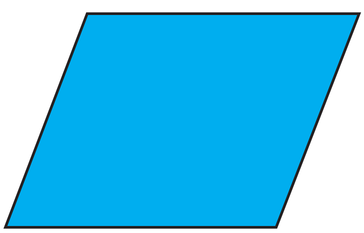
Rectangle

- Opposite sides equal
- Opposite sides parallel
- All angles right
- Diagonals equal
- Diagonals bisect each other



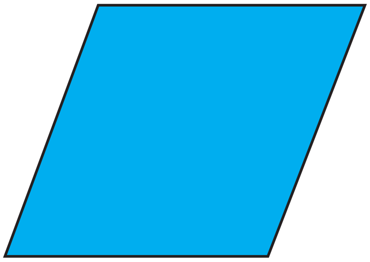
Square

- All sides equal
- Opposite sides parallel
- All angles right
- Diagonals equal
- Diagonals perpendicular bisectors of each other



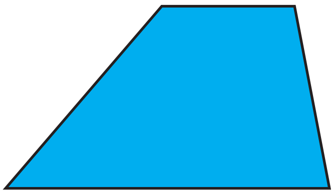
Parallelogram

- Opposite sides equal
- Opposite sides parallel
- Diagonals bisects each other
- Opposite angles equal
- Sum of angles on the same side 180°



Rhombus

- All sides equal
- Opposite sides parallel
- Diagonals perpendicular bisectors of each other
- Opposite angles equal
- Sum of angles on same side 180°



Trapezium

- Only one pair of opposite sides parallel
- Sum of angles on each of non-parallel sides 180°

Isosceles trapezium

- Only one pair of opposite sides parallel
- Non-parallel sides equal
- Diagonals equal
- Angles on each of parallel sides equal
- Sum of angles on each of non-parallel sides 180°

Squares

We have learnt to draw rectangles and squares of specified sides using set squares, in class 5. Just to refresh your memory, draw a square of side 4 centimetres.

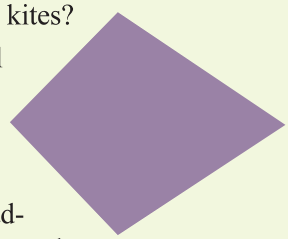
We have seen how we can draw perpendiculars using a compass, in the lesson, **Equal Triangles**. Draw a square using this technique.

Suppose the length of a diagonal is specified, instead of the length of a side?

A Kite that doesn't fly

Have you flown kites?

What is the usual shape of a kite?



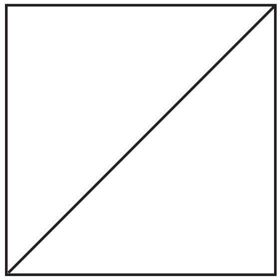
This also is a quadrilateral.

In this, pairs of adjacent sides are equal.

Such a quadrilateral is called a kite in geometry also.

For example, how do we draw a square of diagonal 5 centimetres?

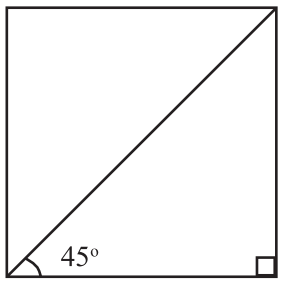
Let's make a rough sketch of a square and its diagonal.



The diagonal splits the square into two triangles. Can you say what the angles of these triangles are?

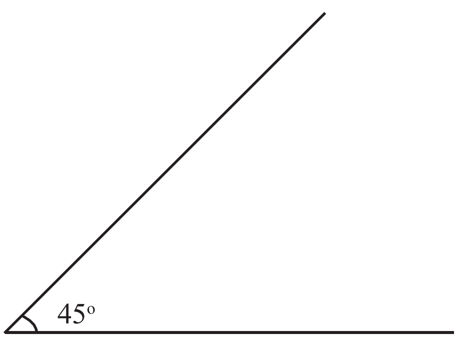
One angle in each is right; and each is an isosceles triangle.

So the other angles are 45°. (How is that?)

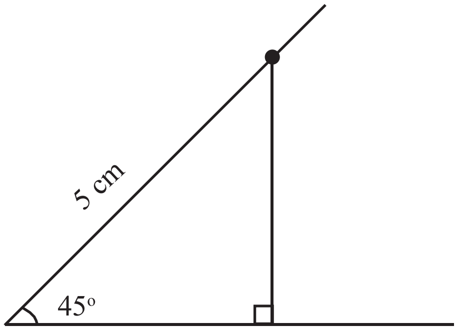


Now can't we draw a square of diagonal 5 centimetres?

First draw a horizontal line and then another line started at 45° at one end;

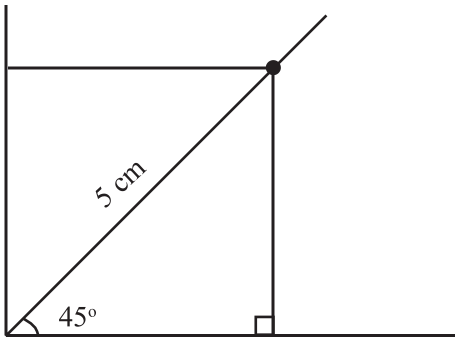


Next mark 5 centimetres on the slanted line and drop a perpendicular line from this point to the bottom line.



(We can draw the perpendicular by making a 45° angle at this point also).

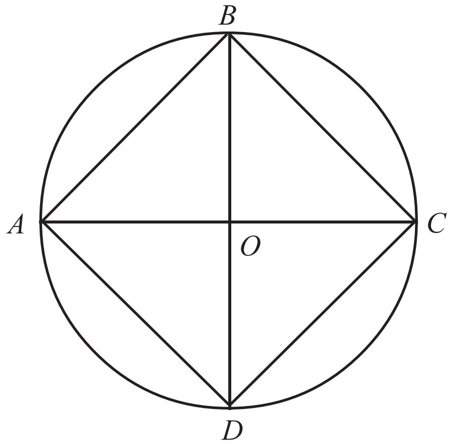
Now we can draw perpendiculars from the two corners and complete the square.



And erase the pieces jutting out to make the figure clean.

There is another way to draw a square.

Draw a circle and two perpendicular diameters. Join their ends.



The four triangles *OAB*, *OBC*, *OCD*, *ODA* are congruent.

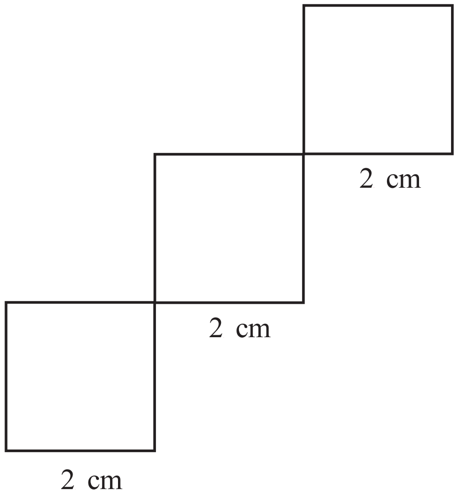
So, what can we say about the quadrilateral *ABCD*?

Doesn't this give another method to draw a square of diagonal 5 centimetres?

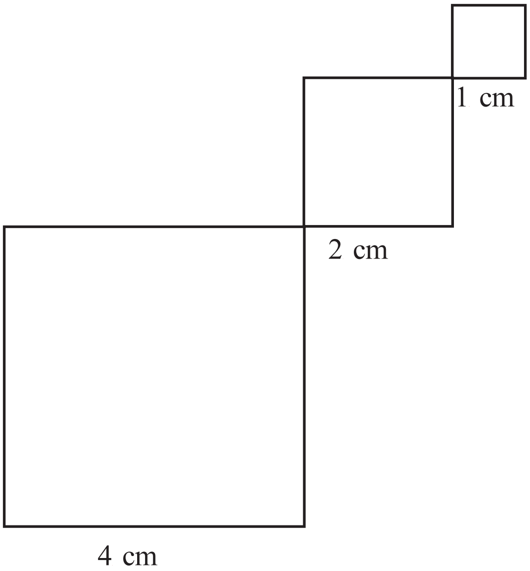
Draw a circle of radius 2.5 centimetres and two perpendicular diameters.

Can you draw these patterns of squares in your notebook?

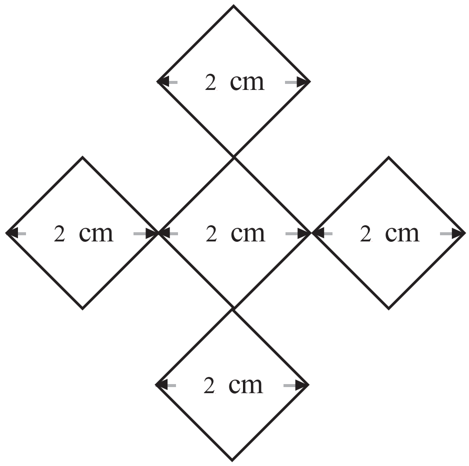
(1)



(2)



(3)



Rectangle

Can't you draw a rectangle of specified sides?

Draw a rectangle of side 8 centimetres and 5 centimetres.

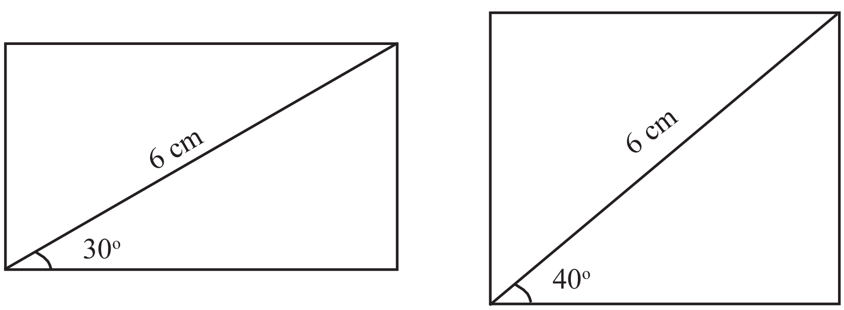
Can we draw rectangles of specified diagonals?

For example, how do we draw a rectangle of diagonal 6 centimetres?

We can draw a square of this diagonal as before. How do we draw a rectangle which is not a square?

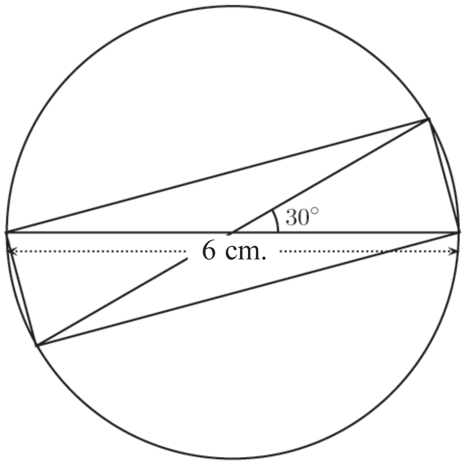
Unlike a square, the angle between a diagonal and a side need not be 45° in other rectangles.

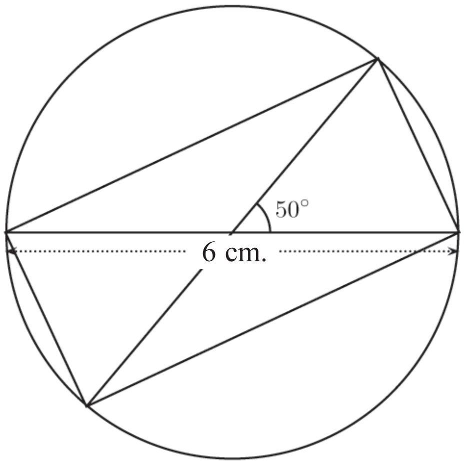
Thus we can draw many rectangles of diagonal 6 centimetres:



We can draw these by first drawing the angle and then perpendiculars, as we did for squares.

We can draw rectangles of specified diagonals by drawing circles also. In rectangles which are not squares, the diagonals are not perpendicular to each other; So we can draw rectangles using any pair of diameters:

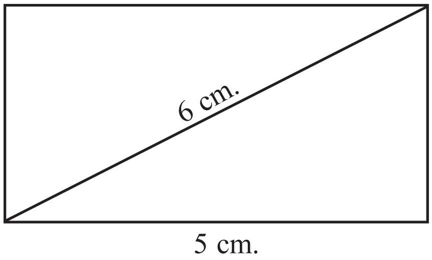




Can you draw a rectangle like this, with diagonal 5 centimetre and 40° angle between them?

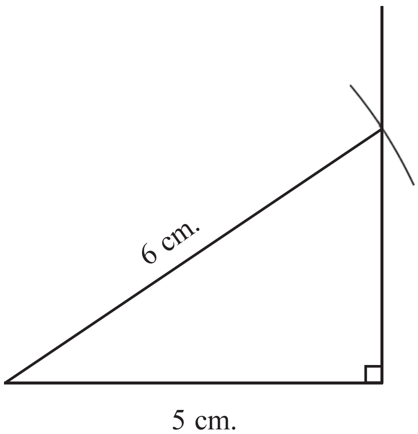
Another question : how do we draw a rectangle of one side 5 centimetres and diagonal 6 centimetres?

To get an idea of how this rectangle would look, let's draw a rough sketch without these actual lengths:



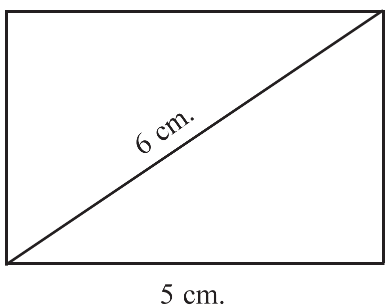
The diagonal splits the rectangle into two right triangles. How about drawing one of these triangles first?

We have to draw a right triangle of hypotenuse 6 centimetres and one of the other sides 5 centimetres:



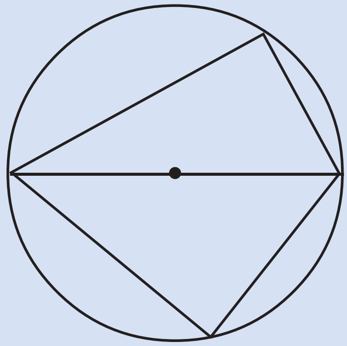
Thus we get half our rectangle.

We can draw the upper half also and complete the rectangle:



Rectangle in circle

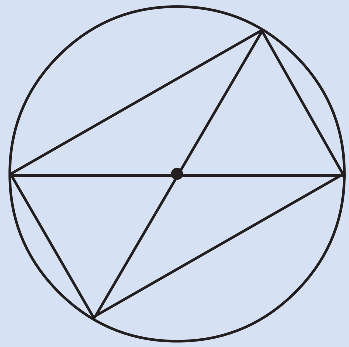
Draw a circle and a diameter. Mark a point in each half of the circle and join them with the ends of the diameter:



The quadrilateral thus get may not be a rectangle. But the angles on either side of the diameter is right. (Why?).

What about the other two angles?

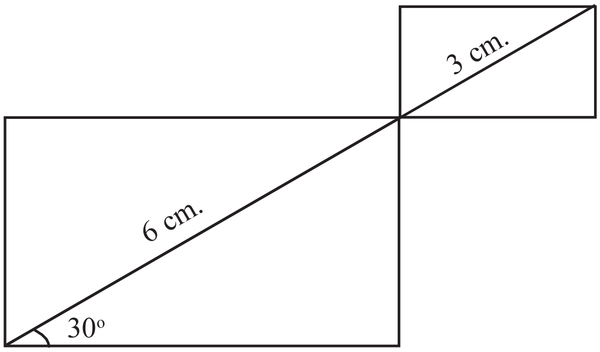
What if the right corners are at the ends of another diameter?



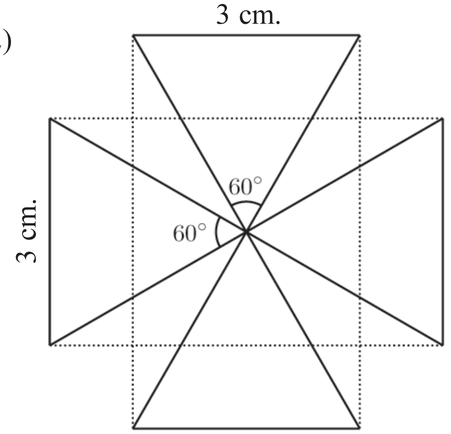
All four angles are right; that is, the quadrilateral is a rectangle.

Draw the figures below in your notebook.

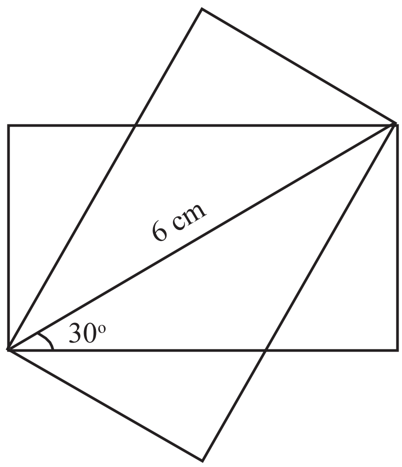
(1)



(2)



(3)



(The rectangles must be equal)

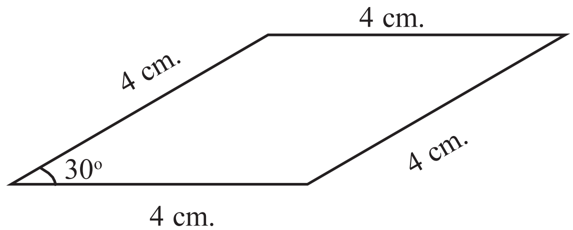
Parallelograms

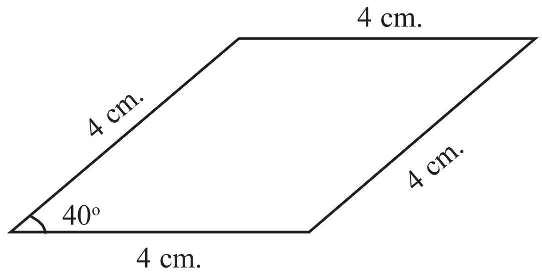
Can you draw a rhombus of sides 4 centimetres? A square is a rhombus and we can easily draw it.

What about a rhombus which is not a square?

Adjacent sides need not be perpendicular.

So we can draw with any angle between them:



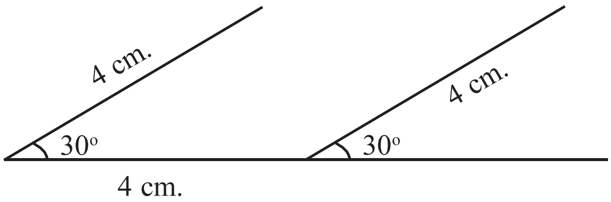


Can you draw the first figure in your notebook?

There are various ways to draw.

First draw a 4 centimetre long line and on its left end, draw another 4 centimetre long line, slanted at 30°. Draw parallels through the other ends of these lines.

Or draw a horizontal line of length 4 centimetres and draw lines of length 4 centimetre at either end, slanted at 30°.



Now we need only to join the upper ends of the slanted lines (and clean the figure by erasing the jutting pieces)

Like this, draw the other rhombus of angle 40° also.

Unlike a square, the diagonals of a rhombus may not be equal. How do we draw a rhombus with diagonals of specified lengths?

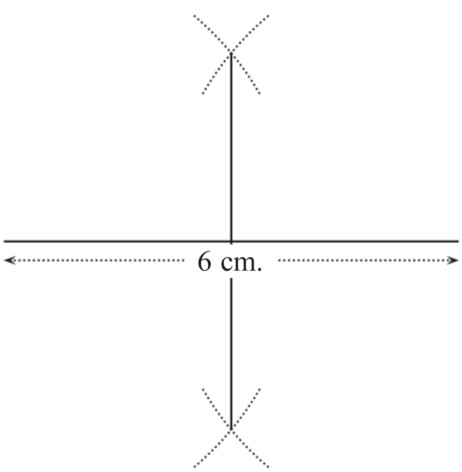
For example, suppose we want to draw a rhombus of diagonals 6 centimetres and 4 centimetres.

This is easy, once we recall that the diagonals of a rhombus are perpendicular bisectors of each other.

First draw a 6 centimetre long line and its perpendicular bisector.

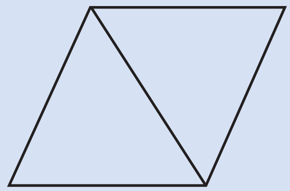
Now mark 2 centimetres on the upper and lower parts of the bisector.

We get our rhombus by joining these points with the ends of the first line.



Isosceles Triangles

A diagonal of a rhombus splits it into two isosceles triangles; and they are equal:



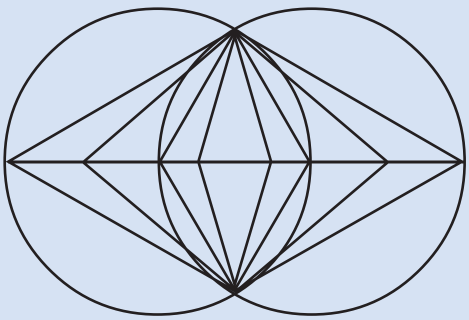
So to draw a rhombus with specified sides and diagonal, we need only to draw isosceles triangles on both sides of the diagonal.

What if the diagonal is equal to a side?

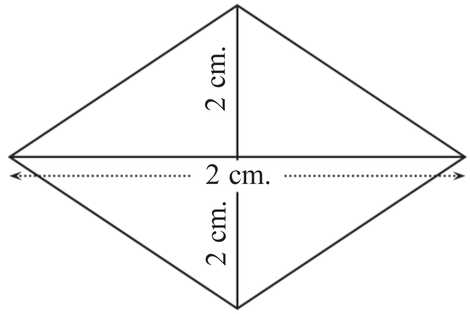
Circles and rhombus

Draw a line and two equal circles centred at its ends. Extend the line to meet the circles.

We can draw several rhombuses with a diagonal along this line.

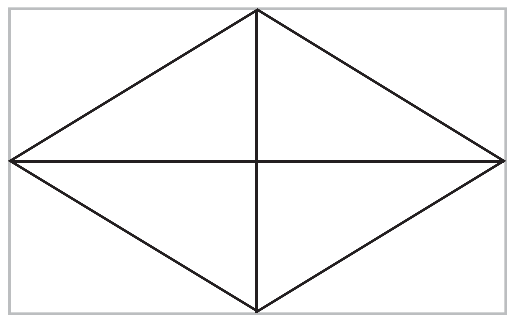


All the four rhombuses in the figure are along the same line, right?



Can we draw this in any other way?

See this picture:

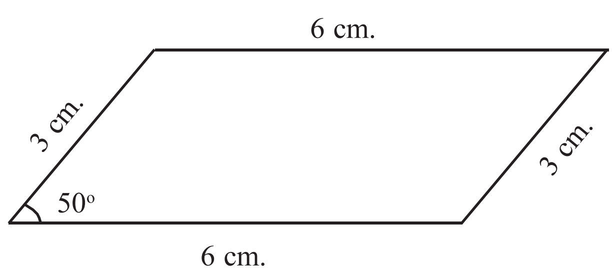


How do we draw a rhombus inside a rectangle?

1) Draw a rhombus of diagonals 5.5 centimetres and 3 centimetres in your notebook.

2) Draw also a rhombus of diagonals 5.5 centimetres and 3.5 centimetres.

We can draw a parallelogram, which is not a rhombus, by specifying some measurements. For example, see this figure:



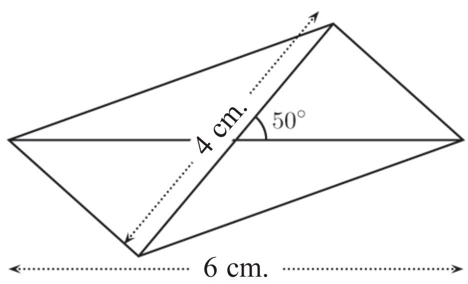
As in the case of a rhombus, we can first draw a 50° angle of sides 6 centimetres and 5 centimetres and then parallels through their ends.

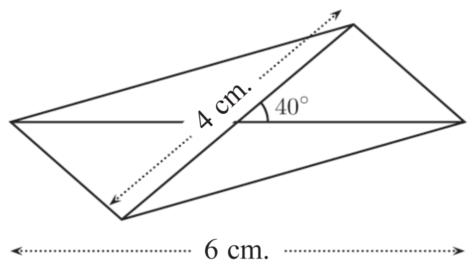
Or we can draw a 6 centimetre long line, then 3 centimetre long lines at its ends slanted at 50° and finally join the ends of the slanted lines

Try it!

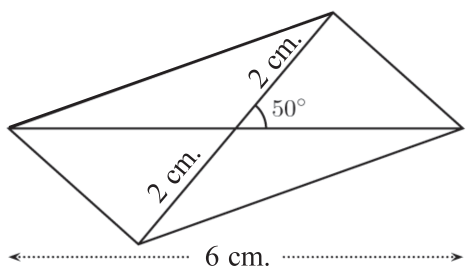
Draw also a parallelogram with sides of these lengths but slanted at 60°

In parallelograms of unequal sides also, the diagonals bisect each other; but not at right angles. So we can draw several parallelograms of same diagonals. See these figures:





We can draw these also, as we drew rhombuses. For the first, instead of drawing the perpendicular bisector of a 6 centimetre long line, we draw a line though the midpoint, slanted at 50°:

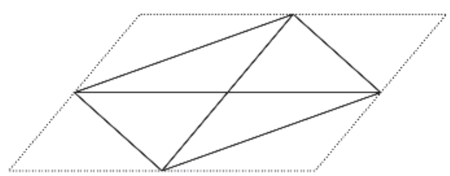


Draw the second figure like this, in your notebook

The diagonals of a parallelogram are not in general equal, and so the length of a side and a diagonal does not specify it completely. (Recall that these were sufficient to determine a rectangle.)

Another way

See this figure:



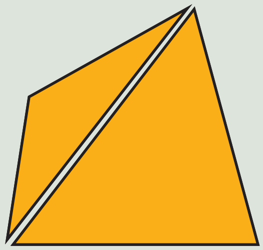
What is the relation between the sides of the outer parallelogram and the diagonals of the inner parallelogram? And what about the angles?

What is the relation between the vertices of the inner parallelogram and sides of the outer parallelogram?

Doesn't this give another method to draw a parallelogram of specified diagonals and angle between them?

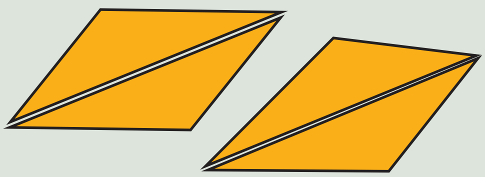
Triangles and quadrilaterals

We can split any quadrilateral into two triangles by drawing a diagonal.



Putting this in reverse, we can join any two triangles with one pair of sides equal, to make a quadrilateral.

If the whole triangles are equal, we can make a parallelogram or a kite.

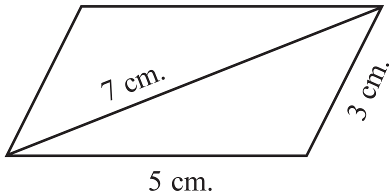


Find out the specialities of the triangles joined, to get various kinds of quadrilaterals.

Suppose the length of two sides and one diagonal are specified?

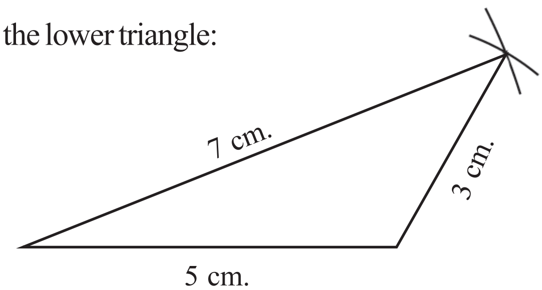
For example, how do we draw a parallelogram of sides 5 centimetres, 3 centimetres and one diagonal 7 centimetres?

Let's fist draw a rough sketch and mark these measurements:

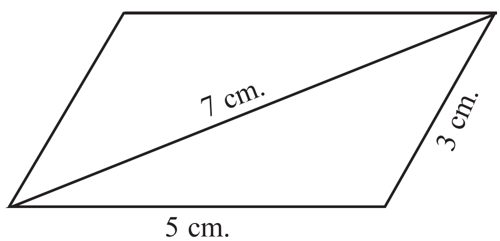


How about drawing the upper and lower triangles separately as we did for a rectangle?

First, the lower triangle:



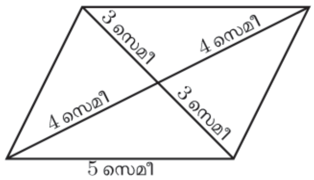
Now we can find the fourth vertex also, by drawing parallel lines or arcs:



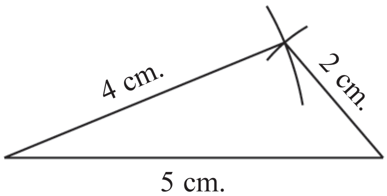
Instead of specifying two sides and a diagonal, suppose it is the other way round.

For example, how do we draw a parallelogram of one side 5 centimetres and diagonals 6 centimetres and 8 centimetres?

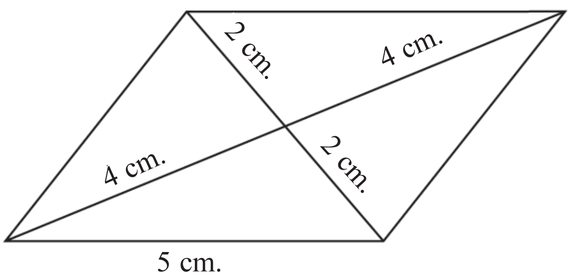
Draw a rough sketch and mark these measurements. Since the diagonals bisect each other, we can write like this:



First, let's draw the triangle formed by the bottom side and half the diagonals above it:



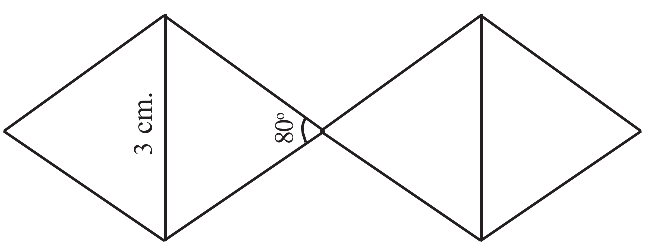
Now we can double the upper sides and complete the parallelogram.



Like this, draw a parallelogram of one side 6.5 centimetres and diagonals 8 centimetres and 7 centimetres.

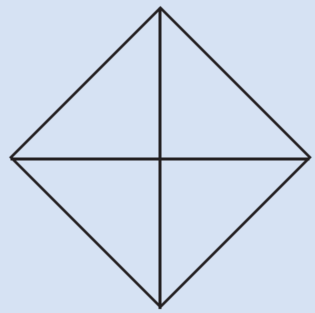
Draw these figures

1) Two equal rhombuses:

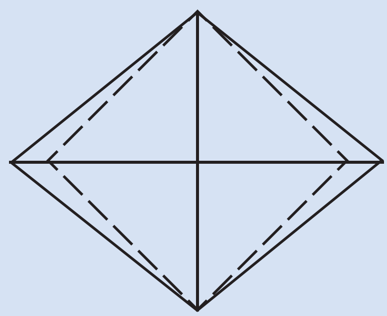


Perpendicular diagonals

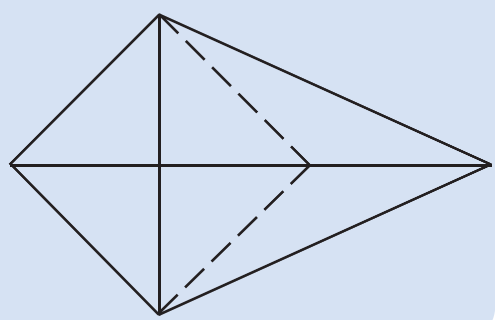
Draw two lines of equal length, bisecting each other at right angles. Joining their ends gives a square:



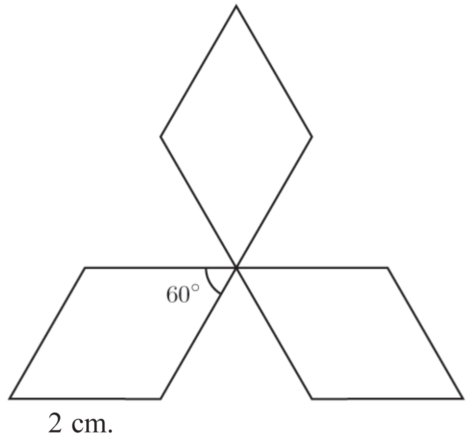
Now extend one line equally to both sides. What do we get on joining the end points?



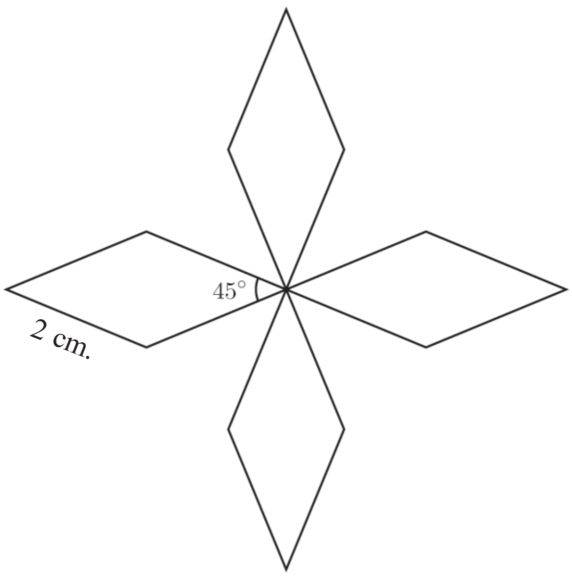
In the first figure, suppose we extend one line to just one scale. What do we get on joining the end points?



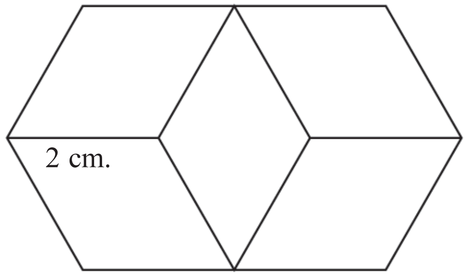
2) Three equal rhombuses:



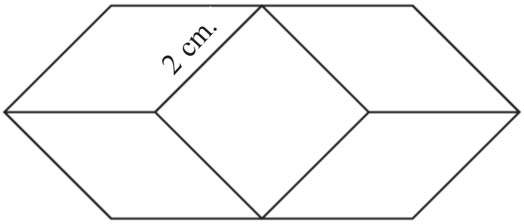
3) Four equal rhombuses:



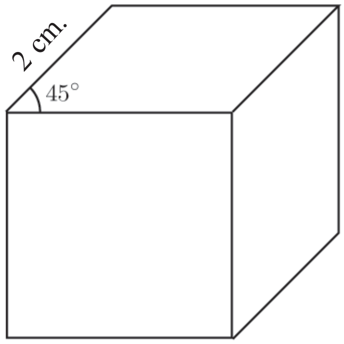
4) Five equal rhombuses:



5) Four rhombuses around a square:

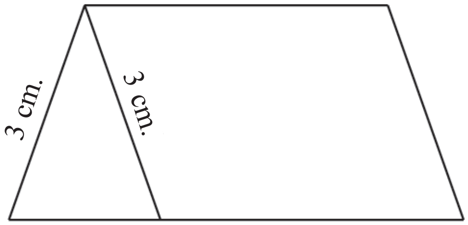


6) Parallelograms on two sides of a square:



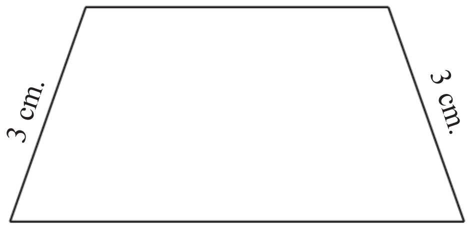
Trapeziums

The figure below shows an isosceles triangle and a parallelogram joined together:



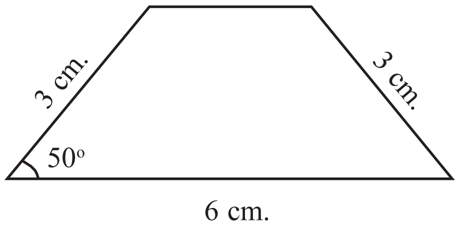
Can't you draw this?

If we erase the joining line, what do we get?



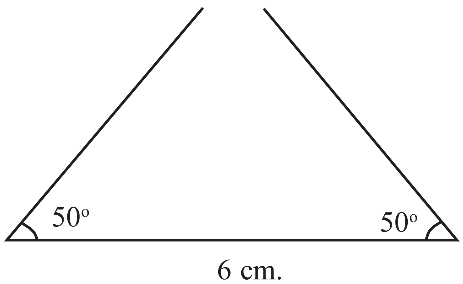
We have drawn a parallelogram of adjacent sides 6 centimetres, 8 centimetres and the angle between them 50°.

Can you draw an isosceles trapezuim of these specifications?

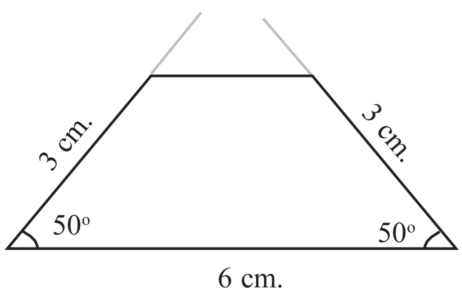


Since it is an isosceles trapezuim, the other angle on the bottom side is also 50°.

So, we can start by drawing a 6 centimetre long line and 50° angles at each end:



Marking 3 centimetres on these lines and joining the ends, we get our trapezium:

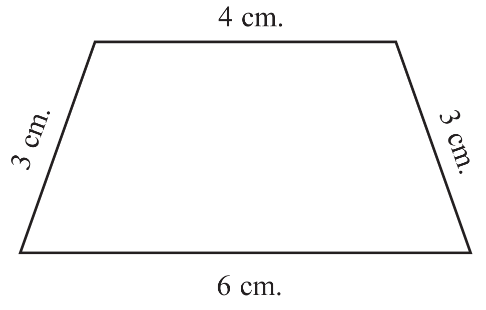


(Can you prove that the top side is parallel to the bottom side?)

Draw an isoceles trapezium with sides of these lengths, but the angle between them 60°.

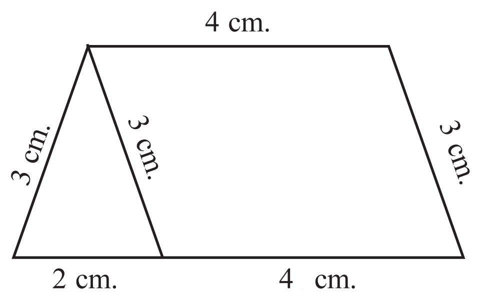
Suppose instead of the angles, the length of the fourth side is specified?

For example, how do we draw the trapezium shown below?

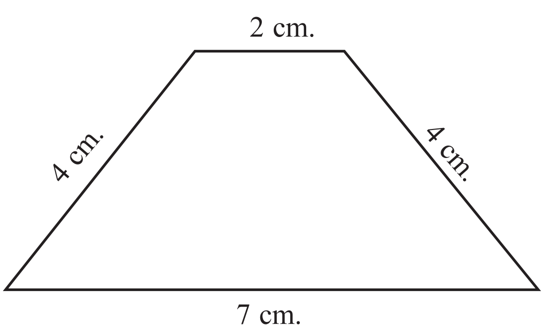


This is a figure we had already drawn.

We drew it by joining an isosceles triangle and a parallelogram:



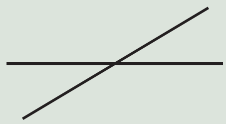
Like this, can you draw in your notebook, the trapezium shown below?



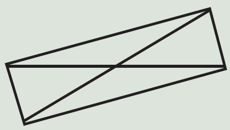
First draw a triangle and then a parallelogram.

Diagonal Changes

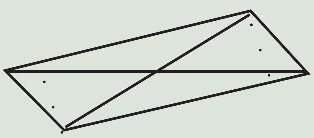
Draw two lines of equal length bisecting each other, but not at right angles;



What kind of quadrilateral do we get on joining their ends?

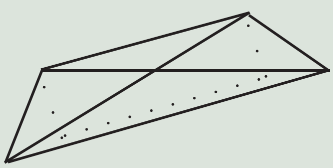


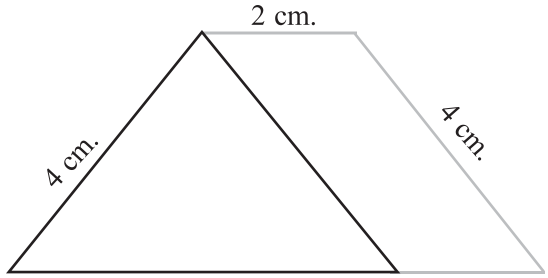
Now as before, suppose we extend one line equally to both sides and join the ends:



What kind of quadrilateral do we get?

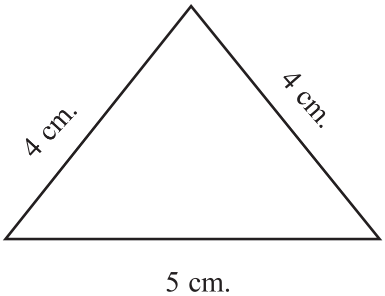
In the first figure, suppose we equally extend the horizontal line to the right and the slanted line downwards, what do we get?



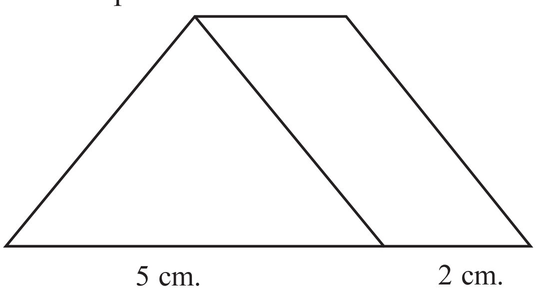


The bottom side of the triangle is $7 - 2 = 5$ centimetres. What about the right side?

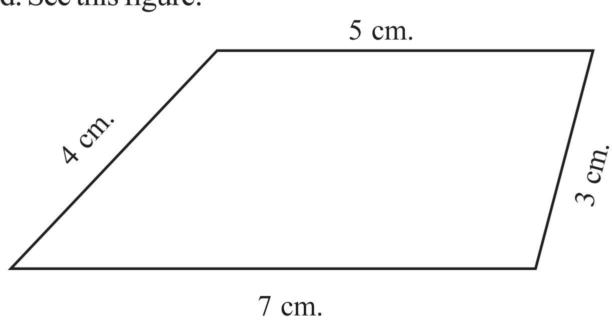
So, we draw a triangle of sides 5 centimetres, 4 centimetres, 4 centimetres:



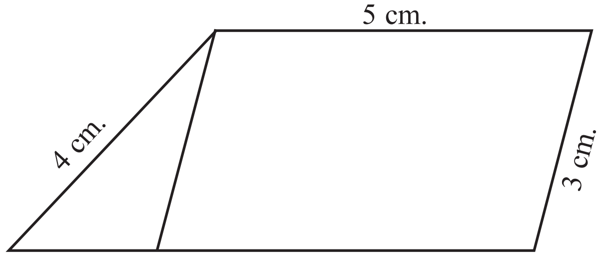
Now extending the bottom side and drawing parallels, we can draw the trapezium:



We can draw non-isosceles trapezuims also like this; all sides must be specified. See this figure:



This also we can split into a triangle and a parallelogram:



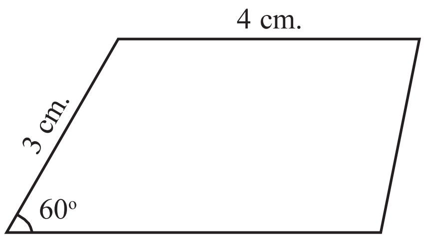
What are the lengths of the other two sides of the triangle?

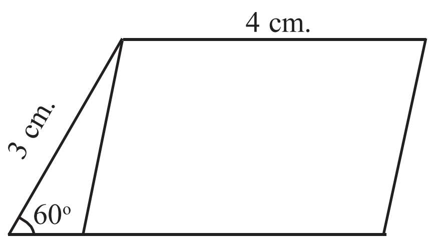
So, we first draw a triangle of sides 2 centimetres, 4 centimetres, 3 centimetres and then a parallelogram as before to make the trapezium. Try!

Instead of four sides, suppose three sides and an angle are specified.

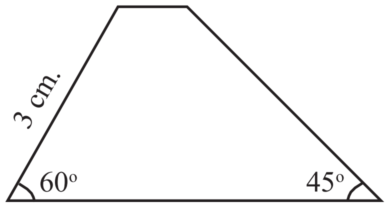
If the measurements are as in the figure alongside, it is not difficult to draw it.

As before, we can first draw a triangle and then a parallelogram:





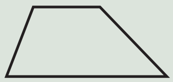
What about two sides and two angles?



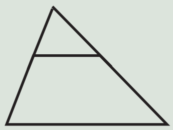
First draw a 5 centimetre long line and angles of 60° and 45° at its ends. Mark 3 centimetres on the left line and draw a line parallel to the bottom line. Try it. (We can draw the parallel to the bottom line by drawing an angle of 120° at the top end of the left lines.)

Trapezium and triangle

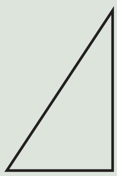
Draw a trapezium



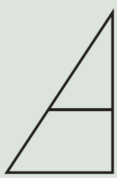
If we extend the pair of non-parallel opposite sides, they met at a point, and we get a triangle.



Now let's start with a triangle.



Draw a line inside, parallel to one of the sides.

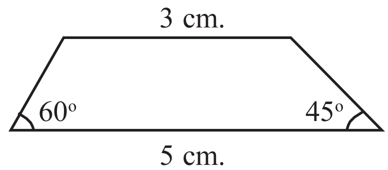


If we erase the portion of the sides above this line, we get a trapezium.

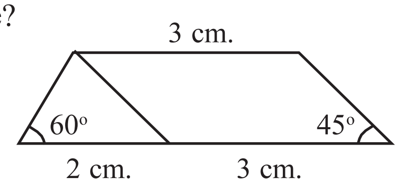
If we start with an isosceles trapezium, what kind of triangle so we get?

On the other hand, cutting an isosceles triangle as above, what kind of trapezium do we get?

How about this trapezium?



Suppose we divide this into a triangle and parallelogram as before?

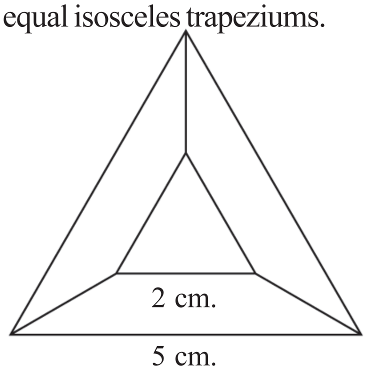


We know the bottom side of the triangle and the angle at one end of it. What about the angle at the other end?

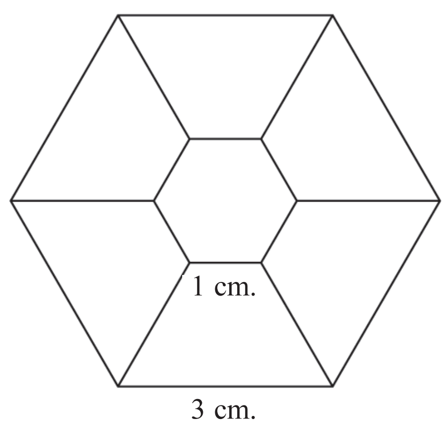
Now can't you draw the triangle and then the trapezium?

Draw the figures below:

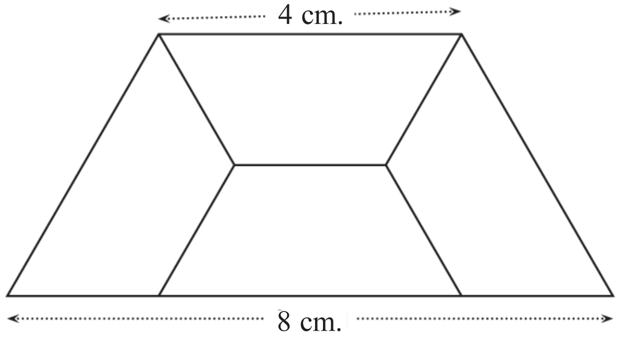
1) Three equal isosceles trapeziums.



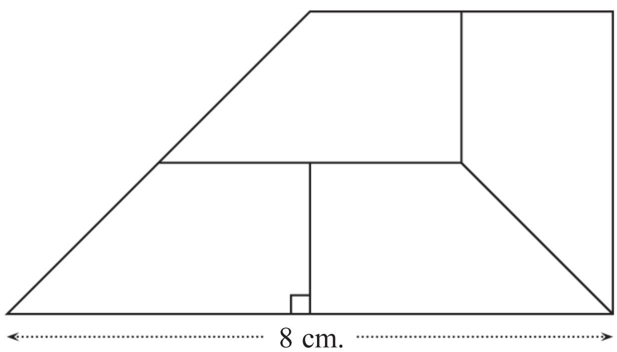
2) Six equal isosceles trapeziums.



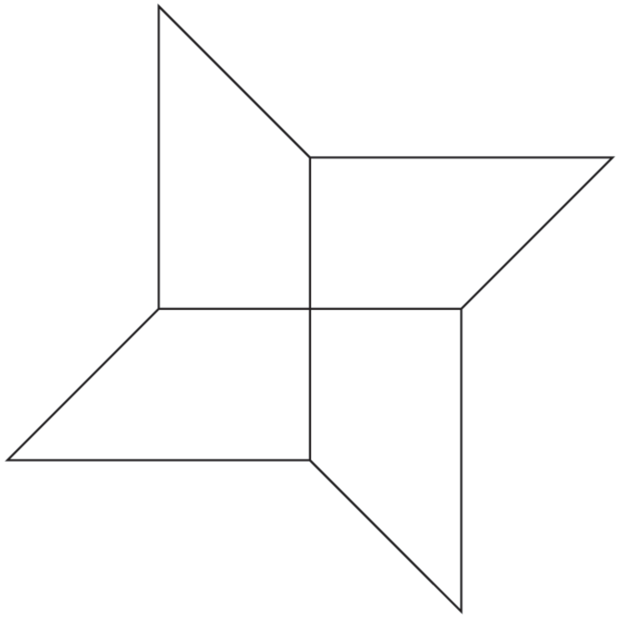
3) Four equal isosceles trapeziums:



4) Four equal trapeziums of another kind:

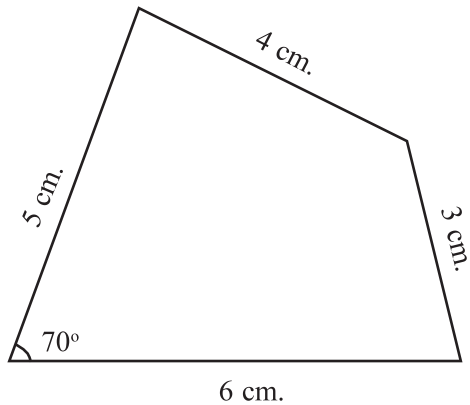


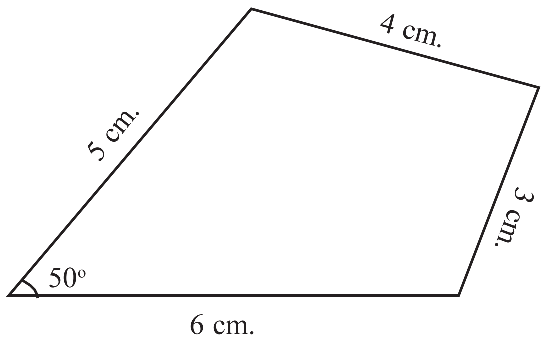
5) Another arrangement of the trapeziums in the last figure:



Quadrilaterals

Now let's draw plain quadrilaterals without any specialities. Two quadrilaterals may not be equal, even if all four sides are the same. So, we can draw different quadrilaterals of same sides. See these quadrilaterals:



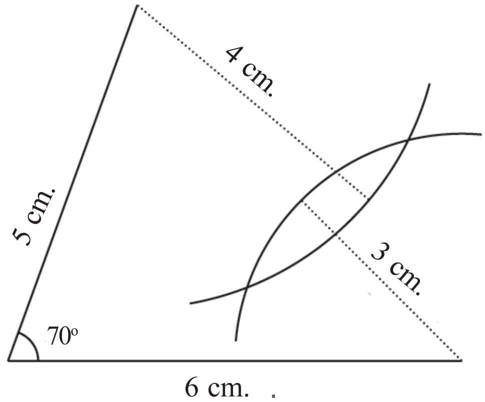


Can you draw these in your notebook?

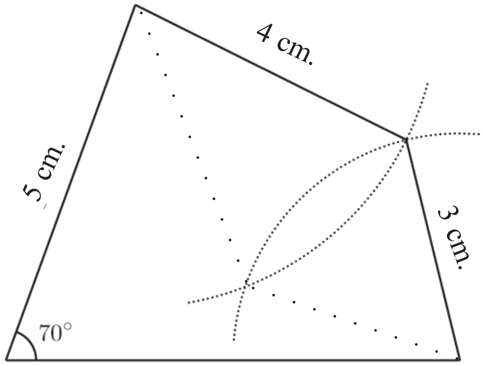
Let's see how we draw the first one. Draw a 6 centimetre long line and then a 5 centimetre line at one of its ends, slanted at 70°.

Now we have three vertices of the quadrilateral. How do we locate the fourth vertex?

It is 4 centimetres from the top vertex and 3 centimetres from the right vertex. Thus it is on the two circles centered at these vertices and of radii equal to these lengths.



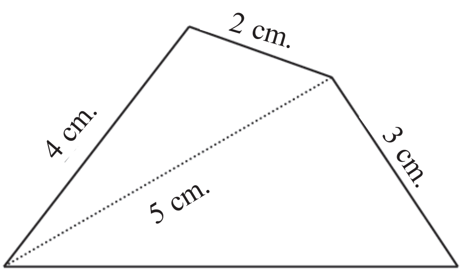
Taking one point of intersection of these circles we can draw our quadrilateral.



(And we don't take into account the depressed quadrilateral got by taking the other point)

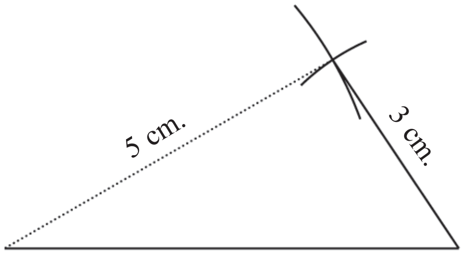
Like this, try to draw the other quadrilateral with 50° angle.

Instead of four sides and an angle, four sides and a diagonal also determines a quadrilateral.



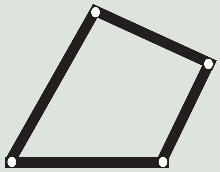
How do we draw this?

First draw the lower triangle:



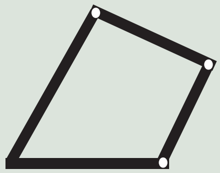
Unstable quadrilateral

Cut out these strips of plastic or thick card-board of lengths 3, 4, 5, 6 centimetres and join their ends using pins or thumbtacks to make a quadrilateral.

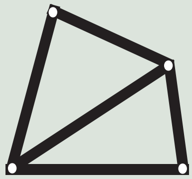


We can move the sides to get different quadrilaterals; and all of them have the same sides.

Now remove the pin from one corner and paste the ends of these sides real fast. Can you move the sides now?

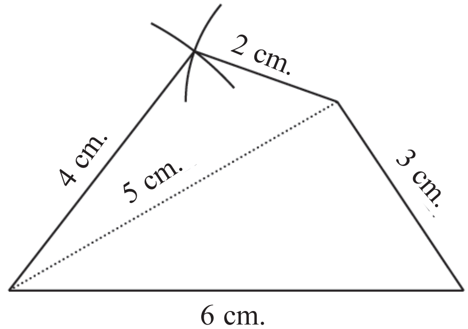


Instead of pasting two ends together, suppose we fit a cross piece like this:

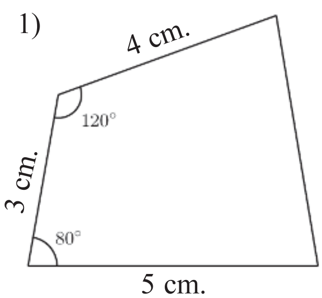


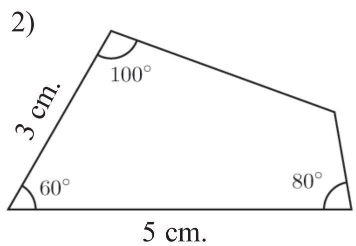
Can you move the sides now?

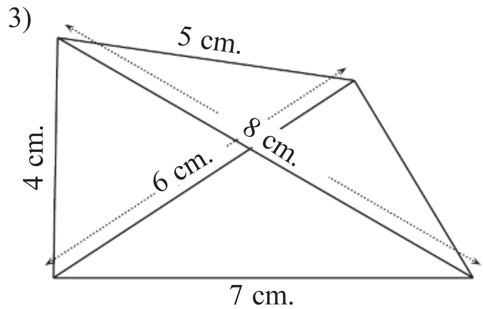
Draw the upper triangle also, we get the quadrilateral.



Draw the quadrilaterals shown below.





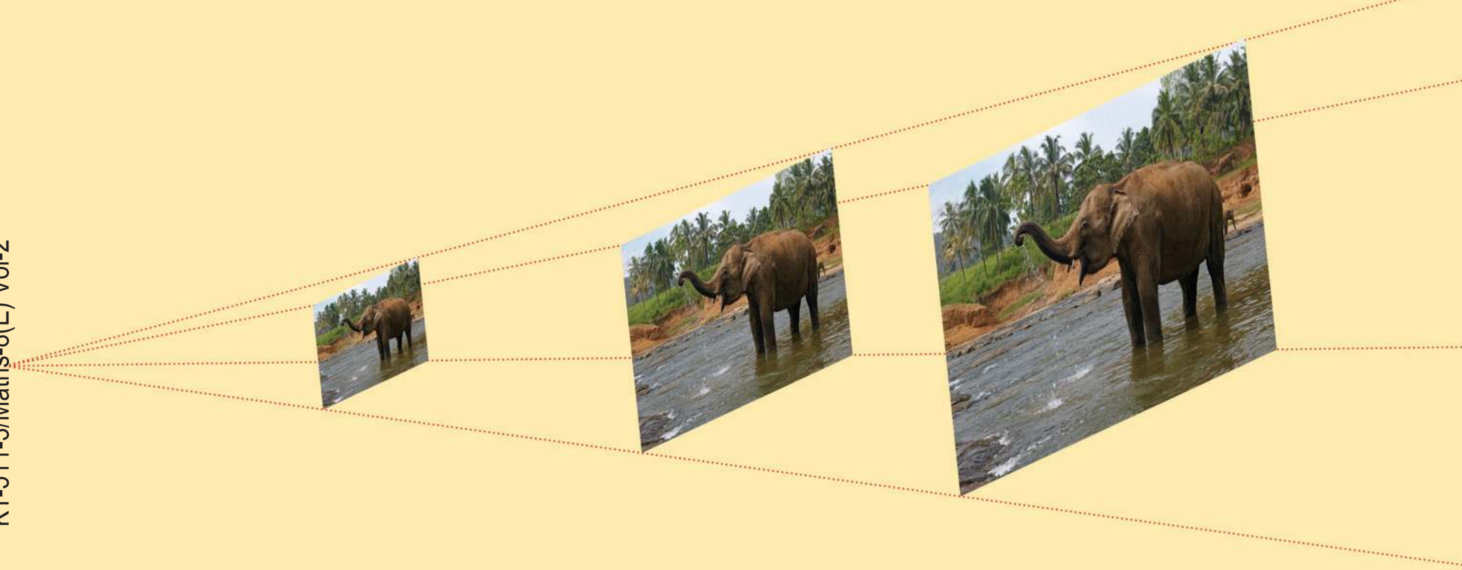


Looking back

Learning outcomes	What I can	With teacher's help	Must Improve
• Explaining various methods of drawing squares.			
• Drawing rectangles by different methods.			
• Finding the measurements needed to specify a parallelogram.			
• Drawing parallelogram according to specification.			
• Finding the measurements needed to specific trapezium			
• Explaining the methods of drawing trapezium according to specification.			
• Determining the measurements needed to specify a quadrilateral.			

7

Ratio



KT-511-3/Maths-8(E) Vol-2

Part relations

See this picture:

The line AB is divided into five equal parts.

Let's call the first three parts together as AP. In what all ways can we describe the relation between the lengths of the lines AP and BP?

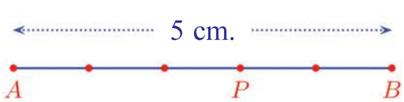
- The relations of AP and BP with AB.
  - AP is $\frac{3}{5}$ of AB
  - BP is $\frac{2}{5}$ of AB
- The relation between AP and BP.
  - BP is $\frac{2}{3}$ of AP
  - AP is $\frac{3}{2}$ of BP
- The relation between AP, BP and the natural numbers 2 and 3.
  - 2 times AP and 3 times BP are equal.
  - $\frac{1}{3}$ of AP and $\frac{1}{2}$ of BP are equal; AP is 3 times this length and BP is 2 times this length.

How do we say all these relations together?

The lengths AP, BP are in the ratio 3 : 2.

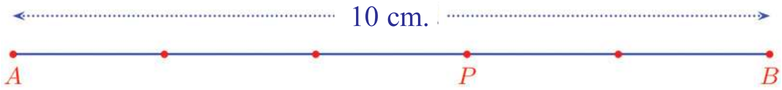
Here, the actual length of AB is not mentioned.

It is 5 centimetres in the picture below:



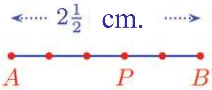
So, the length of AP is 3 centimetres and the length of BP is 2 centimetres.

Suppose we double the length of AB:



The length of AP becomes $2 \times 3 = 6$ centimetres and the length of BP becomes $2 \times 2 = 4$ centimetres. But all the relations above are still true.

What if we halve the length of AB?



Now $AP = 3 \times \frac{1}{2} = 1\frac{1}{2}$ centimeters, $BP = 2 \times \frac{1}{2}$ $= 1$ centimetres.

Again the old relations are not changed.

What does it mean to say that two lengths are in the ratio 3 : 5?

We can't say what the actual lengths are.

It may be 3 centimetres and 5 centimetres.

Or something like

6 centimetres, 10 centimetres

$1\frac{1}{2}$ centimetres, $2\frac{1}{2}$ centimetres

6 metres, 10 metres

It may even be that, measured by some string, the length of the first is 3 lines the string and the length of the second is 5 times the string.

In any case, we can say that the first is 3 times some non specified length and the second is 5 times this length.

Using algebra we can say in general that if we take this length as x centimetres, then the length of the first is $3x$ centimetres and the length of the second is $5x$ centimetres.

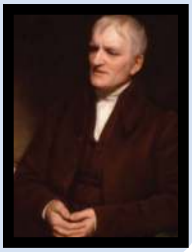
Chemical ratio

In chemistry, substances are classified as elements and compounds.

In the eighteenth century, the scientist Joseph Proust found out that in any compound, the masses of the elements in it are in a definite ratio.

For example, he found out through experiments that in copper carbonate, the mass of copper is 5.3 times that of carbon and the mass of oxygen in 4 times that of carbon. Perhaps it was the thought that such ratios could be expressed as natural numbers using very small particles, which led to the idea of atoms. Such a theory was first put forward by the nineteenth century scientist, John Dalton.

According to his theory, compounds are formed by tiny particles of the elements, called atoms. In any compound, the number of atoms of the elements in it are in a definite ratio.



We can explain ratio of other quantities also like this. For example, the capacities of two bottles are in the ratio 3 : 5 means, using some cup of unspecified size, it takes 3 cups to fill the first bottle and 5 cups to fill the second bottle.

If the capacity of the cup is denoted as x millilitres, we can say in general that the capacity of the first bottle is $3x$ milliliters and the capacity of the second cup is $5x$ milliliters.

What does it mean to say that the number of boys and girls in a class are in the ratio 3 : 5?

If the actual numbers are 30 and 50, we can think of the boys as 3 groups of 10 and the girls as 5 groups of 10.

If the actual numbers are 15 and 25, we can think of the boys and girls again as 3 and 5 groups of 5.

In any case, we can think of the boys as 3 groups and girls as 5 groups, each group with the same number of children.

If the number of children in a group is denoted by x, the number of boys is $3x$ and the number of girls is $5x$.

Elementary ratios

Water is important to sustain life. It is the substance which the human body contains the most.

The elements in water are hydrogen and oxygen. Do you know how much of each water contains? One molecule of water contains 2 atoms of hydrogen and 1 atom of Oxygen. Thus the ratio of hydrogen and oxygen in water is 2:1. The chemical formula for water is H_2O. The elements in common salt that we use are sodium (Na) and chlorine (Cl). They are contained in equal amounts. That is, they are in the ratio 1:1. The chemical formula for common salt is NaCl.

What is the general idea seen in all these examples?

If two quantities are in the ratio *a* : *b*, then there is a quantity *x* such that the first is *ax* and the second is *bx*.

Now let's look at a problem done in class 7. (The section, **Division problem of the lesson, Ratio**)

> The perimeter of a rectangle is 24 metres and its breadth and length are in the ratio 3 : 5. What is the breadth and length?

How did we do this?

We can do this in a different way, using algebra. Since breadth and length are in the ratio 3 : 5, we can take the breadth as $3x$ metres and length as $5x$ metres. To find x, we use the perimeter given in the problem.

Since breadth is $3x$ metres and length is $5x$ metres, the perimeter is

$$2(3x + 5x) = 16x \text{ metres}$$

It is said to be 24 metres. So $16x = 24$ and from this we get,

$$x = \frac{24}{16} = \frac{3}{2}$$

Now can't we calculate the breadth and length?

$$\text{Breadth is} = 3 \times \frac{3}{2} \text{ metres} = 4\frac{1}{2} \text{ metres}$$

$$\text{Length is} = 5 \times \frac{3}{2} \text{ metres} = 7\frac{1}{2} \text{ metres}$$

Let's look at another problem:

The breadth and length of a rectangle are in the ratio 4 : 7. The length is 15 centimetres more than the breadth. What are the length and breadth?

From the given ratio, we see that the breadth is $\frac{4}{11}$ of the sum of bread and length, and the length is $\frac{7}{11}$ of this sum.

So, the difference in length and breadth is $\frac{7}{11} - \frac{4}{11} = \frac{3}{11}$ of the sum. This is said to be 15 centimetres. So the sum of breadth and length is $\frac{11}{3}$ times 15; that is

$$15 \text{ metres} \times \frac{11}{3} = 55 \text{ metres}$$

Now we can calculate, the breadth and length

$$\text{Breadth is} = 55 \times \frac{7}{11} = 35 \text{ metres}$$

$$\text{Length is} = 35 - 15 = 20 \text{ metres}$$

We can do this also using algebra, as in the first problem.

We can take the breadth as $4x$ centimeters and breadth as $7x$ centimeters.

So, length is $7x - 4x = 3x$ centimeters more than the breadth. This difference is given to be 15 centimeters. So, $3x = 15$ and hence $x = 5$.

Now we can compute the breadth and length.

$$\text{Breadth is} = 4 \times 5 \text{ metres} = 20 \text{ metres}$$

$$\text{Length is} = 7 \times 5 \text{ metres} = 35 \text{ metres}$$

Sweet ratio

Do you know the elements in sugar?

Carbon, hydrogen and oxygen. One molecule of sugar contains 12 atoms of carbon, 22 atoms of hydrogen and 11 atoms of oxygen. Thus carbon, hydrogen, oxygen are in the ratio 12 : 22 : 11. The chemical formula of sugar is $C_{12}H_{22}O_{11}$.

What happens when we heat sugar? Why?

One more problem:

The breadth and length of a rectangle are in the ratio 4 : 5 and its area is 320 square metres. What are the breadth and length?

If we take the breadth as $4x$ metres and length as $5x$ metres, area is

$$4x \times 5x = 20x^2 \text{ square metres.}$$

Since we know that this is 320 square metres,

$$20x^2 = 320$$

This means, 20 times the number x^2 is 320.

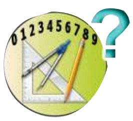
So, this number is $320 \div 20 = 16$. That is,

$$x^2 = 16$$

The number with its square 16 is 4. So, $x = 4$.

$$\text{Breadth is } 4 \times 4 \text{ metres} = 16 \text{ metres}$$

$$\text{Length is } 5 \times 4 \text{ metres} = 20 \text{ metres}$$



In this problem, if we take the breadth as 4 metres and length as 5 metres, the area is 20 square metres; the area given is 16 times this area. Why is it not right to calculate the breadth as 16 times 4 and length as 16 times 5?

1) In a regular polygon, the ratio of the inner and outer angle is 7 : 2. What is each angle? How many sides does the polygon have?

2) The number of girls and boys in a class are in the ratio 7 : 5 and there are 8 more girls than boys. How many girls and boys are there in this class?

3) Blue and yellow paints are mixed in the ratio 2 : 5 to make a new colour. 6 litres more of yellow than blue is taken. How many litres of each is mixed?

4) There are four right triangles, the ratio of perpendicular sides being 3 : 4 in each. One more fact about each is given below. Find the lengths of the sides of each triangle.

(i) The difference in the lengths of the perpendicular sides is 24 metres.

(ii) The hypotenuse is 24 metres.

(iii) The perimeter is 24 metres.

(iv) The area is 24 square metres.

Changing relations

The length of a rectangle is 6 centimetres and its breadth is 4 centimetres.

So, length and breadth are in the ratio 3 : 2.

Suppose we increase the length by 2 centimetres and enlarge the rectangle. Length and breadth are now 8 centimetres and 4 centimetres and their ratio is 2 : 1.

Here's question in reverse:

> The length and breadth of a rectangle are in the ratio 3 : 2. It is enlarged by increasing the length by 2 centimetres and now length and breadth are in the ratio 5 : 8. What are the length and breadth of the original rectangle?

Since the length and breadth of the original rectangle are in the ratio 3 : 2, we take the actual length and breadth as $3x$ centimeters and $2x$ centimetres. When the length is increased by 2 centimetres, these become $3x + 2$ centimetres and $2x$ centimeters. It is said that they are in the ratio 5 : 3. How do we compute x using this fact?

One meaning of saying two quantities are in the ratio 5 : 8 is that 3 times the larger quantity and 5 times the smaller quantity are equal. In our problem, $3x + 2$ is the larger and $2x$ is the smaller. So, the relation between them is

$$3(3x + 2) = 5 \times 2x$$

This can be shortened to

$$9x + 6 = 10x$$

From this, we get $x = 6$ (How?)

To and fro

The sides of a rectangle are 33 centimetres. The sides of another rectangle are 11 centimetres and 6 centimetres. What is the ratio of their perimeters? And the ratio of their areas?
Can you find other pairs of rectangles with such a relation?

Thus the length of the original rectangle is 18 centimetres and breadth is 12 centimetres.

The length and breadth of a rectangle are in the ratio 3 : 2. Increasing the length by any amount, can we make the ratio 4 : 3? How about 5 : 3?

Another problem:

The length and breadth of a rectangle are in the ratio 3 : 2. The length is increased by half as much and the rectangle is enlarged. What is the ratio of the length and breadth of the larger rectangle?

In the original rectangle, the breadth is $\frac{2}{3}$ of the length. When the length is increased by half as much, it becomes $1\frac{1}{2}$ of the original. So, the question is how much times $\frac{2}{3}$ is $1\frac{1}{2}$.

$$1\frac{1}{2} \div \frac{2}{3} = \frac{3}{2} \times \frac{3}{2}$$

$$= \frac{9}{4}$$

Thus in the new rectangle, the length is $\frac{9}{4}$ times the breadth

So, length and breadth are in the ratio 9 : 4.

We can also do this using algebra. Let's start by taking the length and breadth of the original rectangle as $3x$ centimeters and $2x$ centimeters.

Half the length is $1\frac{1}{2}x$ centimeters, adding this length becomes $4\frac{1}{2}x$ centimeters. Breadth remain $2x$ centimeters.

We may say that these are in the ratio, $4\frac{1}{2} : 2$. In terms of natural numbers, it is $9 : 4$.

Ratio and area

Two rectangles have the same perimeter. Length and breadth are in the ratio 2 : 1 in one and 3 : 2 in the other. Which has larger area?

Since perimeters are the same, sum of length and breadth are also same. Taking it as s centimeters, the lengths of the sides of the first rectangle are $\frac{1}{3}s$, $\frac{2}{3}s$ centimeters and the area is $\frac{2}{9}s^2$ square centimeters.

What is the area of the other?

$$\frac{2}{5}s \times \frac{3}{5}s = \frac{6}{25}s^2$$

Which of $\frac{2}{9}$ and $\frac{6}{25}$ is the larger?

$$\frac{2}{9} < \frac{6}{25}.$$

So, the second rectangle has larger area.

Now, what about another rectangle of the same perimeter, but the ratio of sides 1 : 2?

Which has the largest area?

Look at the difference of the sides in all these rectangles. Any connection with the areas?

1) Acid and water are mixed in the ratio 4 : 3 to make a liquid. On adding 10 more litres of Acid, the ratio changed to 3 : 1. How many litres of acid and water does the liquid contain now?

2) Two angles are in the ratio 1 : 2. On increasing the smaller angle by 6° and decreasing the larger angle by 6°, the ratio changed to 2 : 3. What were the original angles?

3) The sides of a rectangle are in the ratio 4 : 5.

   i) By what fraction should the shorter side be increased to make it a square?

   (ii) By what fraction should the longer side be decreased to make it a square?

4) Two quantities are in the ratio 3 : 5.

   (i) If the smaller alone is made four times the original, what would the ratio be?

   (ii) If the smaller is doubled and the larger is halved, what would the ratio be?

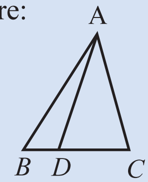
5) (i) (i) The capacities of two bottles are in the ratio 3 : 4. The smaller bottle was filled twice and the larger bottle was filled and emptied into a vessel. Twice the smaller and half the larger was emptied into another. What is the ratio of the quantities of water in the two vessels?

   (ii) In the above problem, what if the capacities of the bottles are in the ratio 4 : 7?

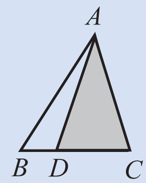
6) The breadth and length of two rectangles are in the ratio 2 : 3. In another rectangle, whose breadth is 1 centimetres less and length is 3 centimetres less than those of the first, this ratio is 3 : 4. Calculate the breadth and length of both rectangles.

Area relations

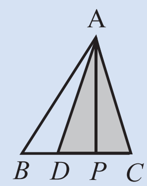
See this figure:



What is the ratio of the areas of ΔABD and ΔACD?



Draw the perpendicular from A to BC.



Taking the length of this perpendicular as h, the area of ΔABD is

$$\frac{1}{2} h \times BD$$

and area of ΔACD is

$$\frac{1}{2} h \times CD$$

So,

$$\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ACD} = \frac{BD}{CD}$$

That is, the ratio of these areas is the same as the ratio of the lengths BD and CD.

So, how do we split a triangle into two triangles of the same area?

What if we want one part to have double the area of the other?

Three quantities

See this picture:

The line AB is divided into 11 equal parts.

AP is 2 such parts together.

PQ is 5 such parts together.

QB is 4 such parts together.

In what all ways can we describe the relations between these pieces.

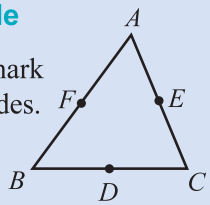
- Relations AP, PQ, QB have with AB
  - AP is $\frac{2}{11}$ of AB
  - PQ is $\frac{5}{11}$ of AB
  - QB is $\frac{4}{11}$ of AB
- Relations between AP, PQ, QB taken in pairs.
  - PQ is $\frac{5}{2}$ times AP; AP is $\frac{2}{5}$ of PQ.
  - QB is $\frac{4}{5}$ of PQ, PQ is $\frac{5}{4}$ times QB.
  - AP is $\frac{2}{4} = \frac{1}{2}$ of QB, QB is $\frac{4}{2} = 2$ times AP.
- Relations AP, PQ, QB have with the numbers 2, 5, 4.
  - 5 times AP and 2 times PQ are equal.

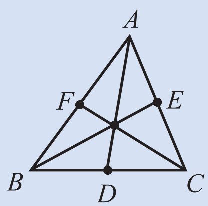
    4 times PQ and 5 times QB are equal.

    2 times AP is equal to QB.
  - $\frac{1}{2}$ of AP, $\frac{1}{5}$ of PQ and $\frac{1}{4}$ of QB are equal. AP is 2 times, PQ is 5 times and QB is 4 times this length.

Centre of a triangle

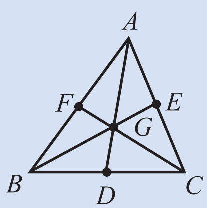
Draw a triangle and mark the midpoints of its sides.





Join each mid point to the opposite vertex.

These lines are called medians of the triangle. Don't they pass through a single point inside the triangle?



This point is called the centroid of the triangle.

The centroid divides each median in the ratio 2 : 1. Thus in our picture,

$$\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = 2$$

The centroid has another property. Cut out a cardboard figure like this. We can balance it at this point on the tip of a pencil.

Thus the centroid of a triangle is its centre of gravity.

As in the case of two quantities, here also we can combine all this by saying that AP, PQ, QB are in the ratio 2 : 5 : 4.

Thus when we say that three quantities are in the ratio

3 : 4 : 2 it means that the least is 2 times, the largest is 4 times and the medium sized 3 times some unspecified quantity.

We can put this in algebra.

If three quantities are in the ratio *a* : *b* : *c*, then there is a quantity *x* such that the first is *ax*, the second is *bx* and the third is *cx*.

Look at this problem:

The sides of a triangle are in the ratio 3 : 5 : 7 and its perimeter is 45 centimetres. What are the length of the sides?

Quantities are in the ratio 3 : 5 : 7 means, they are $\frac{3}{15}, \frac{5}{15}, \frac{7}{15}$ of the sum. In this problem, the sum of the length is the perimeter; that is, 45 centimetres. So, the length of the sides are

$$45 \text{ centimetres} \times \frac{3}{15} = 9 \text{ centimetres}$$

$$45 \text{ centimetres} \times \frac{5}{15} = 15 \text{ centimetres}$$

$$45 \text{ centimetres} \times \frac{7}{15} = 21 \text{ centimetres}$$

We can also do this using algebra. Taking the sides as $3x$, 5x and $7x$ centimeters, the perimeter is $15x$ centimetres.

Right triangles

What is the speciality of a triangle with sides 3 centimetres, 4 centimetres and 5 centimetres? Since $3^2 + 4^2 = 5^2$, it is a right triangle.

Suppose we double each side. Do we get a right triangle?

We also have $6^2 + 8^2 = 10^2$.

Thus the triangle got by doubling the sides is also right. What if we take x times each side?

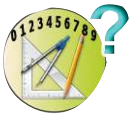
$(3x)^2 + (4x)^2 = 9x^2 + 16x^2 = 25x^2 = (5x)^2$.

Thus a triangle with sides $3x$, $4x$, $5x$ is also right.

In short, any triangle whose sides are in the ratio 3 : 4 : 5 is right.

What about triangles with sides in the ratio 5 : 12 : 13?

Perimeter is given to be 45 centimetres. So we have $15x = 45$, which gives $x = 3$. Thus the lengths of the sides are $3 \times 3 = 9$ centimetres, $5 \times 3 = 15$ centimetres and $7 \times 3 = 21$ centimetres.



Can any triangle have sides in the ratio 3 : 5 : 8?

Let's look at another kind of problem:

In triangle ABC, the sides AB, BC are in the ratio 2 : 3 and the sides BC, CA are in the ratio 4 : 5. What is the ratio of all three sides together?

AB, BC are in the ratio 2 : 3 means,

AB is $\frac{2}{3}$ of BC.

BC, CA are in the ratio 4 : 5 means,

CA is $\frac{5}{4}$ times BC.

So, measured by BC, the length of AB is $\frac{2}{3}$, the length of BC is 1 and the length of CA is $\frac{5}{4}$.

What if we measure by $\frac{1}{12}$ of BC?

All lengths get multiplied by 12.

That is, length of AB is $\frac{2}{3} \times 12 = 8$, the length of BC is 12, and the length of CA is $\frac{5}{4} \times 12 = 15$.

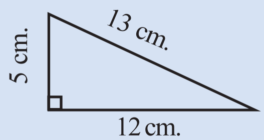
Ratio of sides 8 : 12 : 15

We can do this using algebra also.

The first ratio given means, AB is 2 times some length and BC is 3 times this length. What about the second ratio?

United triangles

A triangle with sides 5 centimetres, 12 centimetres, 13 centimetres is right:



Can we join any triangle with sides in the ratio 3 : 4 : 5 to this to make a large triangle? How many such triangles are there? What are the lengths of their sides?

Concrete mixture

To make concrete, for every sack of cement, two sacks of sand and for every sack of sand, two sacks of gravel are taken. How many sacks of gravel are needed for each sack of cement? For two sacks of sand, four sacks of gravel must be taken. Thus for each sack of cement, two sacks of sand and four sacks of gravel must be taken.

We can put it like this.

The ratio of cement to sand and sand to gravel are both 1 : 2. Writing the second ratio as 2 : 4, we can easily see that cement, sand and gravel are in the ratio 1 : 2 : 4.

BC is 4 times a length and CA is 5 times this length. The length of BC is not the same when measured by these little lengths; so these lengths are also different. Taking them as x and y centimeters,

AB = $2x$ centimetres, BC = $3x$ centimetres

BC = $4y$ centimetres, CA = 5y centimetres

Both $3x$ and $4y$ refer to the length of BC.

So, $3x = 4y$, which gives.

$$y = \frac{3}{4}x$$

Thus we find

$$CA = 5y \text{ cm.} = 5 \times \frac{3}{4}x \text{ cm.} = \frac{15}{4}x \text{ cm.}$$

and hence

$$AB = 2x \text{ cm.}$$
$$BC = 3x \text{ cm.}$$
$$CA = \frac{15}{4}x \text{ cm.}$$

Another way

AB and BC are in the ratio 2 : 3 means, AB is $\frac{2}{3}$ of BC. BC and CA are in the ratio 4 : 5 means CA is $\frac{5}{4}$ times BC.

This can be put like this: measured by $\frac{1}{3}$ of BC, the length of AB is 2; and measured by $\frac{1}{4}$ of BC, the length of CA is 5. What if we measure by $\frac{1}{12}$ of BC? The length of AB would be 8 and the length of CA would be 15. And the length of BC itself is 12. Thus AB, BC, CA are in the ratio 8 : 12 : 15.

This gives the ratio of AB, BC, CA as $2 : 3 : \frac{15}{4}$. In terms of natural numbers, we can write this as 8 : 12 : 15.

1) Johny invested 50000 rupees, Jaleel 40000 rupees and Jayan 20000 rupees to start a business together. They got 3300 rupees as profit in a month, which they divided in the ratio of their investments. How much did each get?

2. The capacities of three water tanks are in the ratio 2 : 3 : 5. The smallest of them can hold 2500 litres. How many litres can the other two hold?

3. The angles of a triangle are in the ratio 1 : 3 : 5. How much is each angle?

4. The outer angles of triangle are in the ratio 5 : 6 : 7. What are the angles?

Angle ratio

The angles of a triangle are in the ratio 1 : 2 : 3.
What are the angles?
What if the ratio is 2 : 3 : 5?
And if 5 : 7 : 12?

Note anything common to these triangles?
And the numbers in each ratio?

5. The sides of a triangle are in the ratio 2 : 3 : 4. The longest side is 20 centimetres more than the shortest side. Calculate the length of all three sides.

6. A box contains beads of three colours. Black beads and white beads are in the ratio 3 : 5. White and red are in the ratio 2 : 3. What is the ratio of all three colours?

7. The length, breadth and height of a rectangle block are in the ratio 3 : 2 : 5 and its volume is 3750 cubic centimeters. Calculate the length, breadth and height.

Looking back

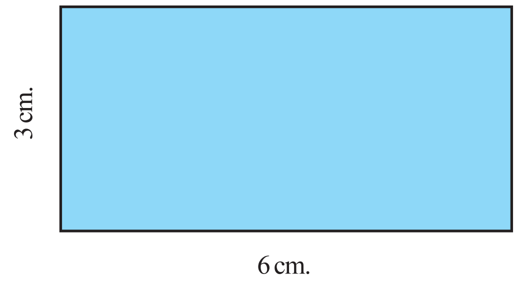
Learning outcomes	What I can	With teacher's help	Must Improve
• Interpreting the ratio of two quantities as parts and times.			
• Given the ratio of two quantities and the one of them, computing the other.			
• Given the ratio of two quantities and some relation between them, computing both quantities.			
• Interpreting the ratio of three quantities in various ways.			
• Computing the ratio of three quantities from their ratios in pairs.			
• Given the ratio of three quantities and some relation between two of them, computing the quantities.			

8

Area of Quadrilaterals

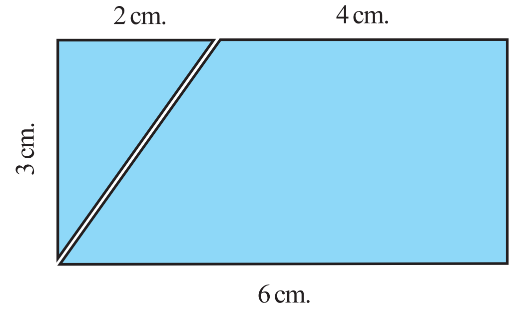
Same Area

See this rectangle:

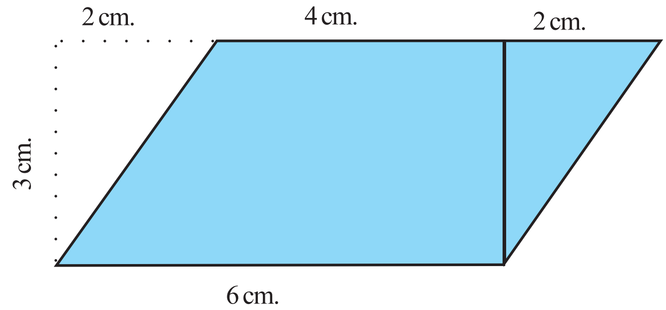


What is its area?

Now cut out this rectangle in thick paper. Cut off a triangle from the left as shown below;



What if we attach this triangle on the right?



Now we have a parallelogram. (Can you prove that it is indeed a parallelogram?)

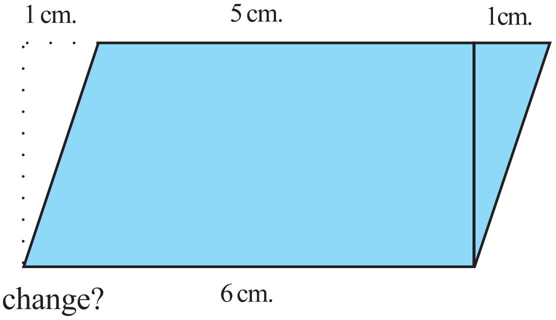
What is the area of this parallelogram?

We have not removed anything from the rectangle;

Only rearranged a piece.

So, the area of the parallelogram is also 18 square centimetres.

What if we cut out a triangle, taking 1 centimetre instead of 2 centimetres at the top?



Does the area change?

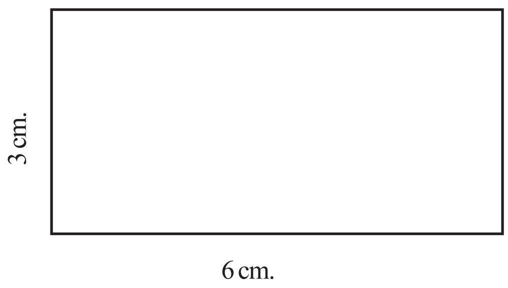
How about 3 centimetres?

All parallelograms got like this have area 18 square centimetres and one side 6 centimetres; the other sides area different.

So we can ask;

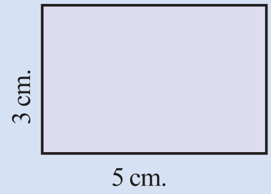
Can we draw a parallelogram of sides 6 centimetres and 4 centimetres with area 18 centimetres?

Let's start with the rectangle seen first:

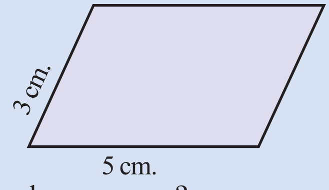


Changing areas

Draw a rectangle of sides 5 centimetres and 3 centimetres.



Now draw a parallelogram of the same measurements with slanted sides.

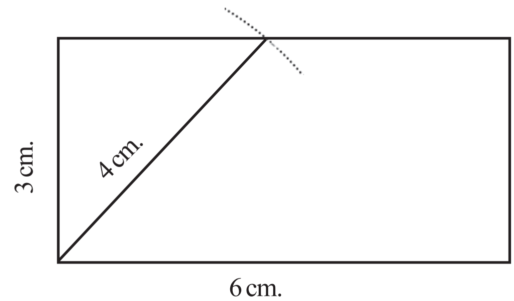


Is its area less or more?

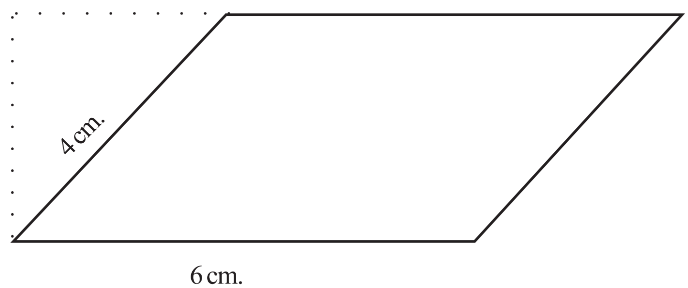
We want a parallelogram with the second side 6 centimetres.

For that, draw an arc of radius 4 centimetres centred at the bottom corner and mark the point where it cuts the top side.

Draw a line joining the bottom corner with this point:



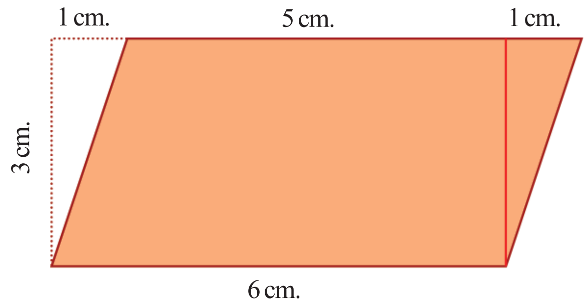
Now from the other bottom corner, draw a line parallel to this line and extend the top side to meet it:

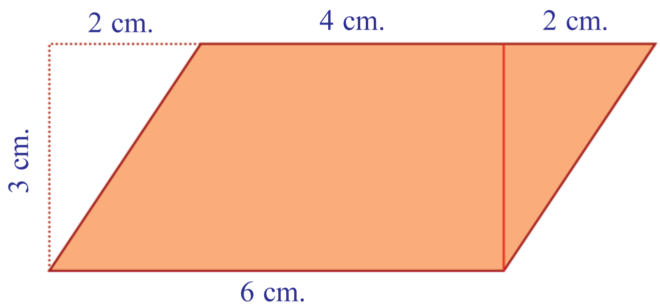


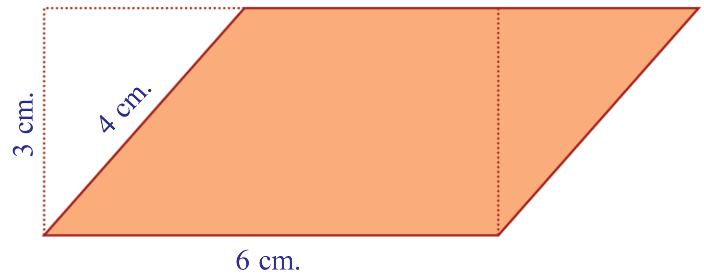
Can you draw like this, a rhombus of sides 6 centimetres and area 18 centimetres?

Parallelograms

We have drawn many parallelograms with one side 6 centimetres and area 18 square centimetres.





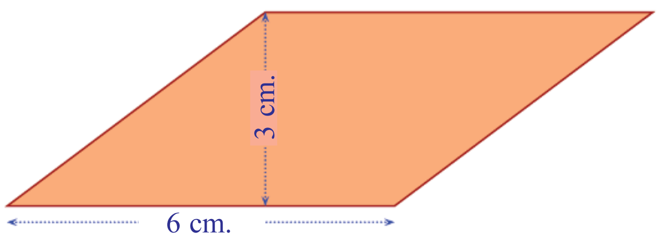


In all these, the second sides are different. But there is another measurement that remains unchanged.

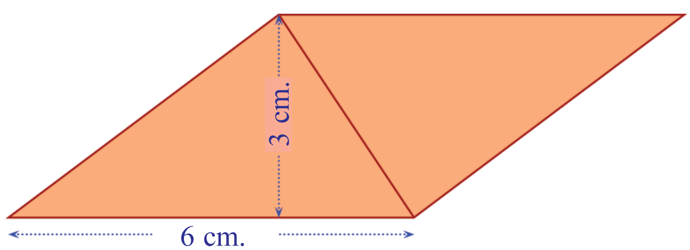
In all, the distance between the top and bottom sides is 3 centimetres, right?

So, for any parallelogram with the length of a pair of parallel sides 6 centimetres and the distance between them 3 centimetres, is the area equal to 18 square centimetres?

See this figure:



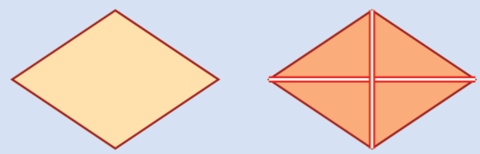
We can draw a diagonal to split it into two triangles:



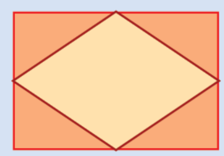
What is the area of the lower triangle?

Doubling

Cut out two equal rhombuses and cut it along the diagonals.



Put the four triangles got around the uncut rhombus like this:



How is the area of this rectangle related to the area of one rhombus?

What can you say about the sides of this rectangle?

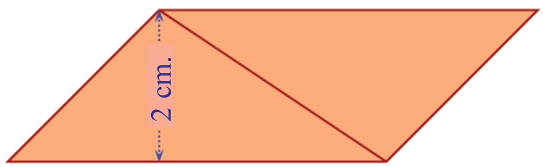
Since the bottom side is 6 centimetres and the distance from the opposite vertex is 3 centimetres its area is $\frac{1}{2} \times 6 \times 3 = 9$ square centimetres.

The area of the upper triangle is also the same. (Why?)

So, the area of the parallelogram is 18 square centimetres.

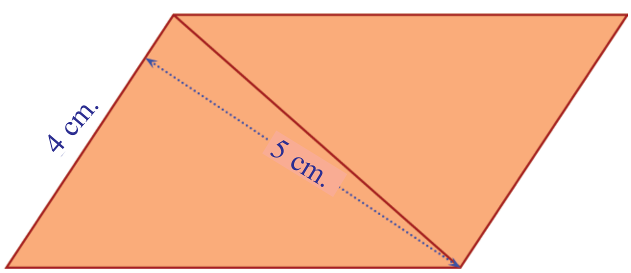
Like this, can we compute the area of this parallelogram?

As before, we draw a diagonal to split it into two triangles:



The area of each triangle is half the product of 5 and 2; and so the area of the parallelogram is this product, that is $5 \times 2 = 10$ square centimetres.

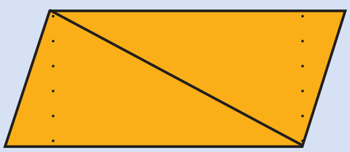
What if the measurements are like this?



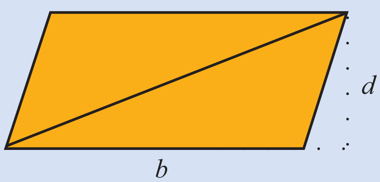
Both triangles have one side of 4 centimetres and the distance from the opposite vertex 5 centimetres; so the area of each is half of 4×5; which means the area of the parallelogram is $4 \times 5 = 20$ square centimetres.

Long diagonal

Just we found the area of a parallelogram by drawing the shorter diagonal, we can also draw the other diagonal and do it.



This longer diagonal also splits the parallelogram into two equal triangles. To find the area of the lower triangle, we need only to draw the perpendicular from the top right corner to the base extended.



Thus the area of one triangle is $\frac{1}{2}bd$.

The area of the parallelogram is

$$2 \times \frac{1}{2}bd = bd$$

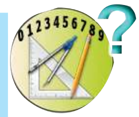
We can compute the area of any parallelogram like this.

The area of a parallelogram is the product of one side with the distance to the opposite side.

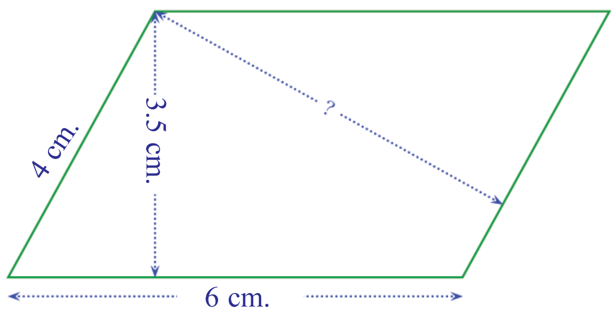
Can we draw a parallelogram of sides 5 centimetres, 6 centimetres and area 35 square centimetres?

What is the maximum area of parallelograms of sides 6 centimetres and 5 centimetres?

What is the speciality of the parallelogram of maximum area?



Another problem. See this parallelogram.



What is the distance between its left and right sides?

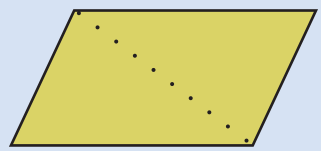
Since the bottom side is 6 centimetres and the distance to the top side is 3.5 centimetres, its area is $6 \times 3.5 = 21$ square centimetres.

Since the left side is 4 centimetres, the distance to the right side is multiplied by 4 should also give the area, 21 square centimetres. Thus the distance to the right side is $21 \div 4 = 5.25$ centimetres.

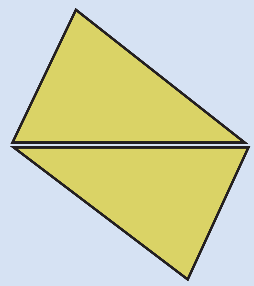
1) Draw a parallelogram of sides 5 centimetres, 6 centimetres and area 25 square centimetres and area 25 square centimetres.
2) Draw a parallelogram of area 25 square centimetres and perimeter 24 centimetres.

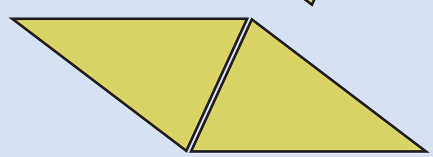
Without changing area

See the parallelogram below:



We can cut along the diagonal and join the pieces along equal sides to make new parallelograms.

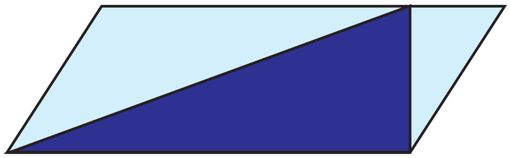




How are the sides and a diagonal of these related to the sides and a diagonal of the original parallelogram?

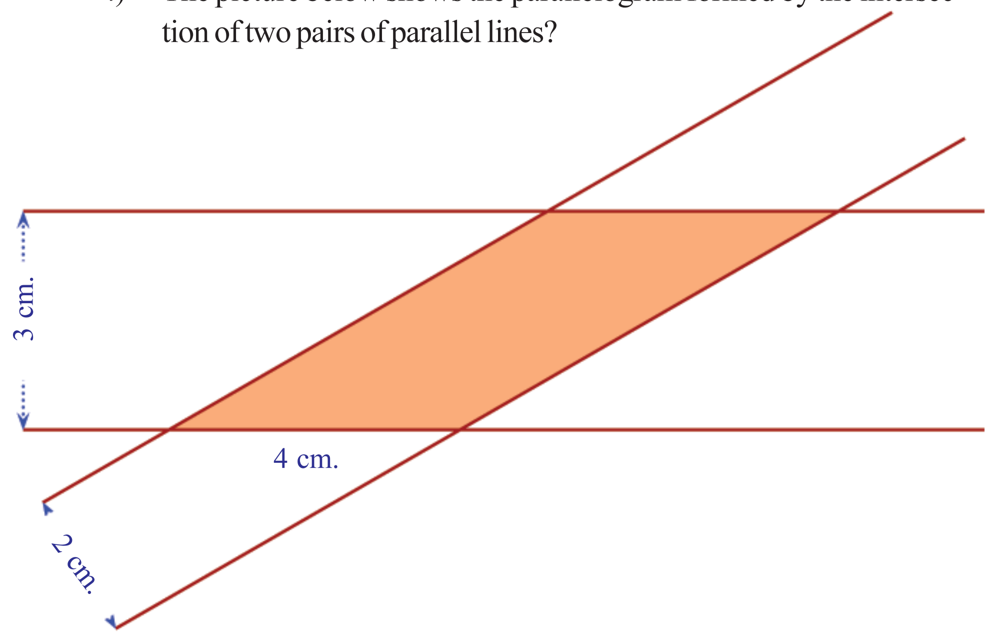
What if we cut along the other diagonal?

3) In the figure, the two bottom corners of a parallelogram are joined to a point on the top side.



The area of the dark triangle in the figure is 5 square centimetres. What is the area of the parallelogram?

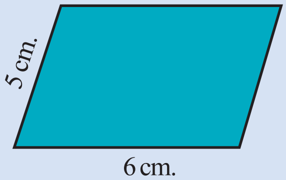
4) The picture below shows the parallelogram formed by the intersection of two pairs of parallel lines?



What is the area of this parallelogram? And the perimeter?

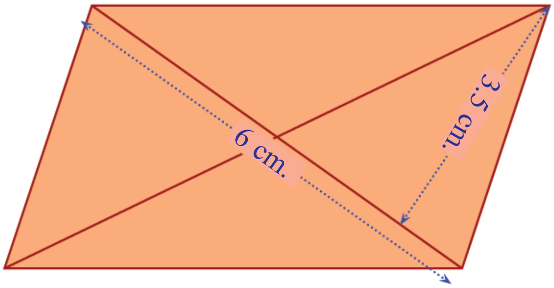
Unchanging area

Draw a parallelogram of sides 6 centimetres and 5 centimetres.



We want to draw another parallelogram with the same length for the top and bottom sides, the sides on the left and right are 10 centimetres and of the same area. How do we do it?

5) Compute the area of the parallelogram below:



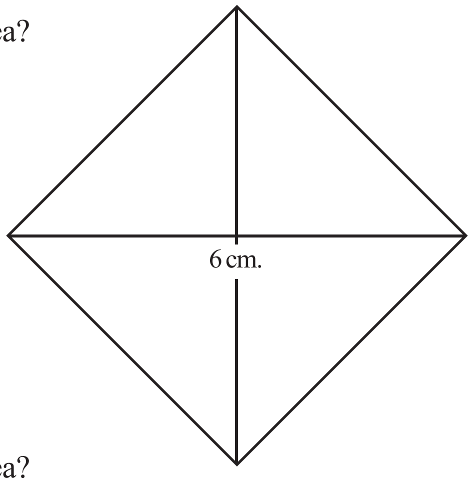
What is the maximum area of parallelograms of diagonals 6 centimetres and 4 centimetres? What is the speciality of the parallelogram of maximum area?

Rhombus

We can draw a square of specified sides; also a square of specified diagonals.

Draw a square of diagonal 6 centimetres.

What is its area?



What is the specality of the parallelogram with the distance between both pairs of parallel sides of the same?

What is its area?

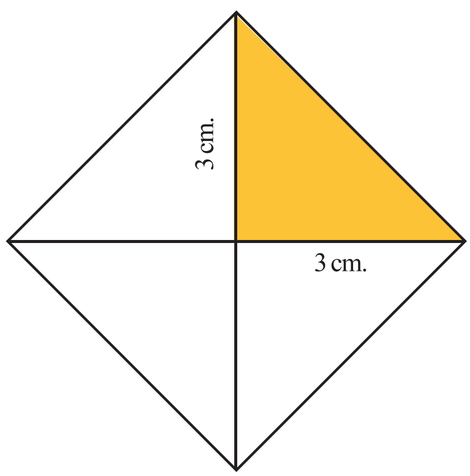
We know that the area of a square is the square of its side; but it is not easy to calculate the side of this square; Instead, we can think like this.

This square is formed by joining four equal, isosceles right triangles.

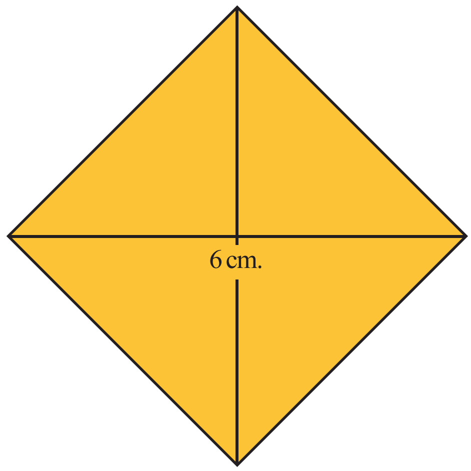
All these triangles have perpendicular sides of length 8 centimetres.

So the area of one triangle is

$\frac{1}{2} \times 3 \times 3 = 4\frac{1}{2}$ square centimetres.



The area of the full square is $4 \times 4\frac{1}{2} = 18$ square centimetres.



Likewise, what is the area of a square of diagonal 5 centimetres?

The sum of the areas of four isosceles right triangles of perpendicular sides $2\frac{1}{2}$ centimetres. That is,

$$4 \times \frac{1}{2} \times 2\frac{1}{2} \times 2\frac{1}{2} = \frac{25}{2} = 12\frac{1}{2} \text{ sq.cm.}$$

To get the general idea, let's use a bit of algebra.

Taking the lengths of diagonals as d, the lengths of the perpendicular sides of all four isosceles right triangles is $\frac{1}{2}d$.

Area of one such triangle is

$$\frac{1}{2} \times \frac{1}{2}d \times \frac{1}{2}d = \frac{1}{8}d^2$$

Area of the square is,

$$4 \times \frac{1}{8}d^2 = \frac{1}{2}d^2$$

In usual language,

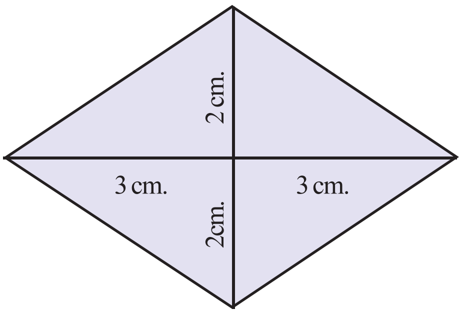
The area of a square is half the square of the diagonal.

According to this, to draw a square of area 8 square centimetres, how long should be the diagonal? Draw it!

The diagonals of a non-square rhombus also splits it into four right triangles (not isosceles though).

So, we can compute the area of any rhombus also like this.

For example, let's look at a rhombus with diagonals, 6 centimetres and 4 centimetres:



Area of the rhombus is

$$4 \times \frac{1}{2} \times 3 \times 2 = 12 \text{ sq.cm.}$$

In general, the area of a rhombus with diagonals d_1 and d_2 is

$$4 \times \frac{1}{2} \times \frac{1}{2} d_1 \times \frac{1}{2} d_2 = \frac{1}{2} d_1 d_2$$

That is,

The area of a rhombus is half the product of the diagonals.

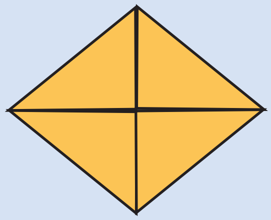
For example, the area of a rhombus with diagonals 5 centimetres and 4 centimetres is 10 square centimetres.

1) Draw a square of area $4 \frac{1}{2}$ square centimetres.
2) Draw a non-square rhombus of area 9 square centimetres.
3) The area of a rhombus is 216 square centimetres and the length of one of its diagonals is 24 centimetres. Compute the following measurements of this rhombus.
   i) Length of the second diagonal
   ii) Length of a side
   iii) Perimeter
   iv) Distance between sides

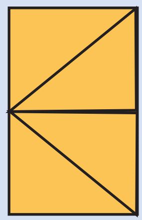
Rhombus and rectangle

Draw a rhombus and its diagonals.

Cut along the diagonals to get four triangles.



These can be rearranged to make a rectangle.

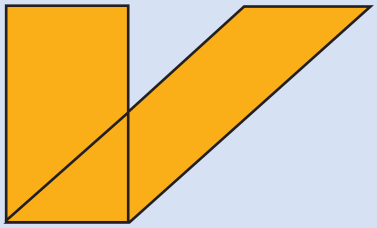


The area of the rectangle is the area of the rhombus. How are the sides of the rectangle related to the diagonals of the rhombus?

So, how is the area of the rhombus related to the lengths of its diagonals?

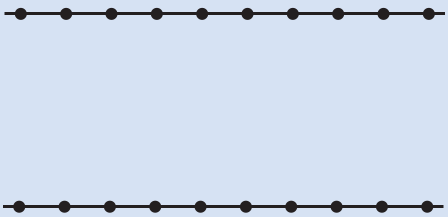
Slanted rectangle

See this figure:



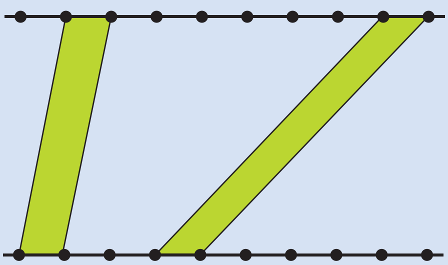
Can you prove that the rectangle and parallelogram have the same area?

Draw a pair of parallel lines and mark equidistant points on each:



Join any two adjacent points at the bottom with two adjacent points at the top. We can draw several such quadilaterals.

Are they all parallelograms?



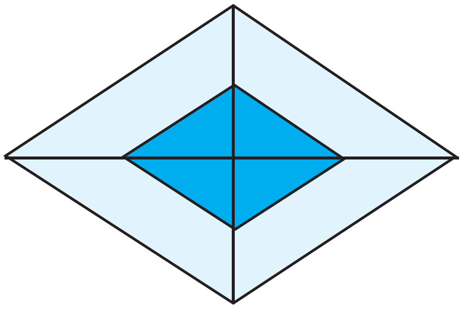
What can you say about their area.

4) A 68 centimetre long rope is used to make a rhombus on the ground. The distance between a pair of opposite corners is 16 metres.

   i) What is the distance between the other two corners?

   ii) What is the area of the ground bounded by the rope?

5) In the figure, the midpoints of the diagonals of a rhombus are joined to form a small quadrilateral:



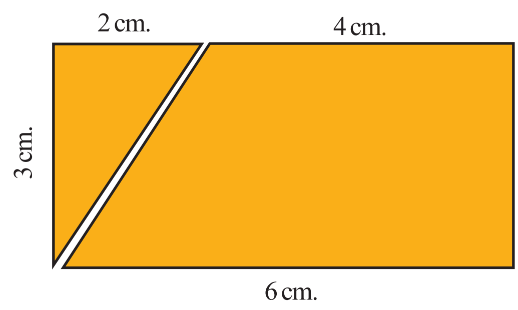
   i) Prove that this quadrilateral is a rhombus.

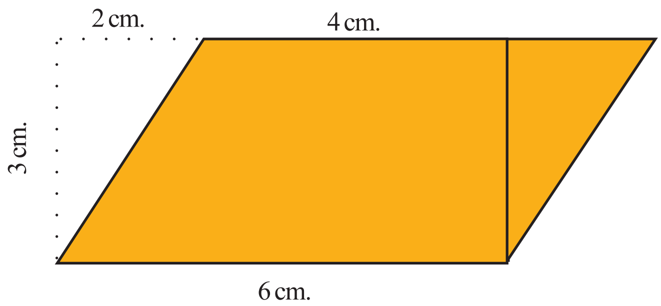
   ii) The area of the small rhombus is 3 square centimetres. What is the area of the large rhombus?

6) What is the area of the largest rhombus that can be drawn inside a rectangle of sides 6 centimetres and 4 centimetres?

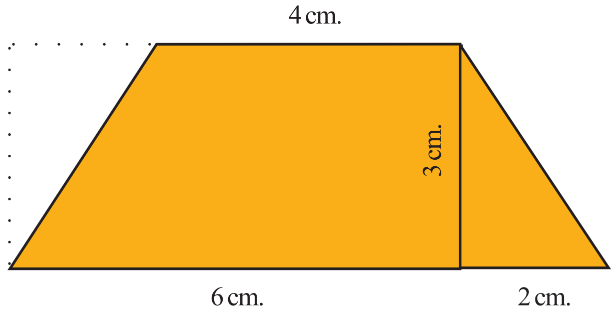
Isosceles trapezium

Cutting a triangle from one side of a rectangle and attaching it to the other side, could we make a parallelogram?





What if we flip the triangle on the right?



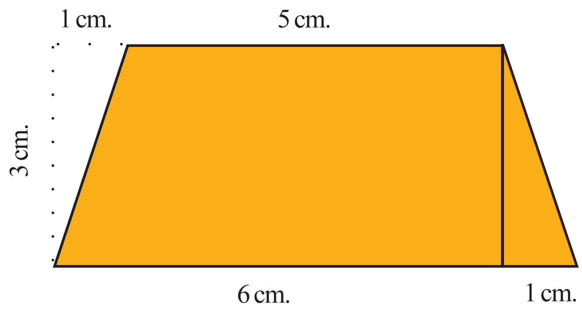
The area of this isosceless trapezium is the same as that of the rectangle; that is, 18 square centimetres.

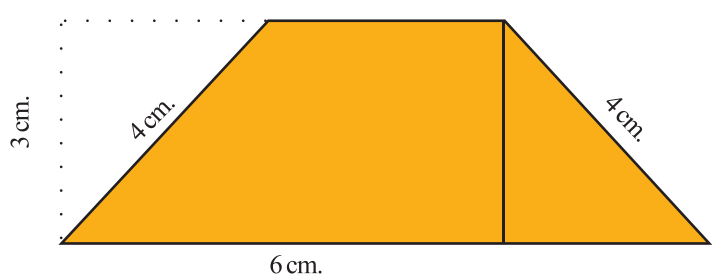
What other lengths of this do we know?

What are the lengths of the parallel sides?

And the distance between them?

As we did for parallelogram, let's cut out triangles of different sizes:





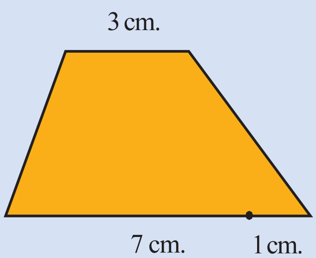
All these isosceles trapeziums are of the same area, 18 square centimetres.

In each, the top side of the rectangle is decreased a bit; and the bottom is increased that much. In other words, in all these, the sum of the parallel sides is the same as that of the rectangle; that is 12 centimetres.

1) Draw a rectangle of sides 7 centimetres and 4 centimetres. Draw isosceles trapeziums of the same area, with the following specifications.
   - i) Lengths of parallel sides 9 centimetres, 5 centimetres.
   - ii) Lengths of non-parallel sides 5 centimetres.

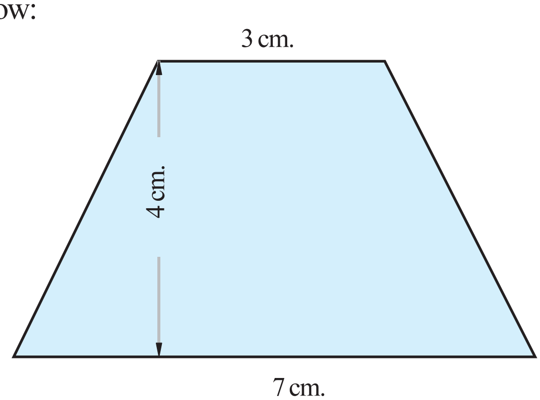
Unchanging area

Draw a parallelogram of sides



We want to draw another parallelogram with the same lengths for the top and of the same area. How do we do it?

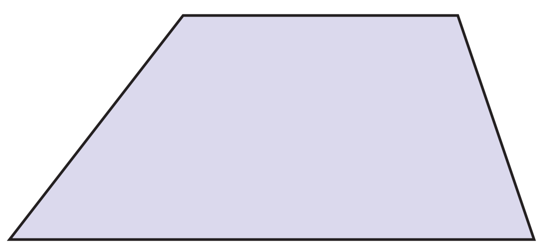
2) Calculate the area of the isosceles trapezium drawn below:



3) The parallel sides of an isosceles trapezium are 8 centimetres and 4 centimetress long; and non-parallel sides are 5 centimetre long. What is its area?

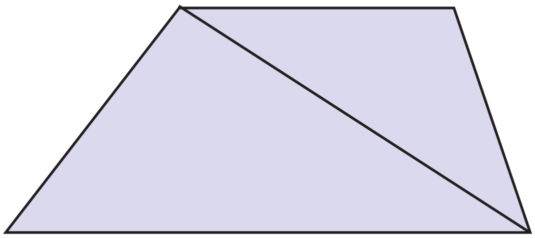
Trapezium

See the a non-isosceles trapezium:

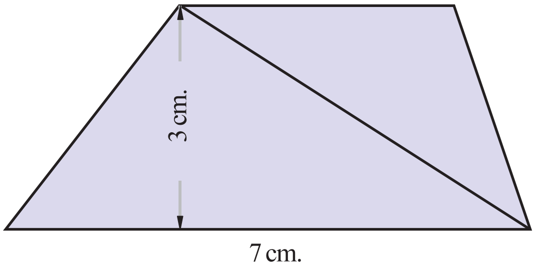


How do we compute its area?

As we did in parallelograms, let's draw a diagonal and split it into two triangles:



To compute the area of the lower triangle, we need, the length of the bottom side and distance to the opposite vertex:

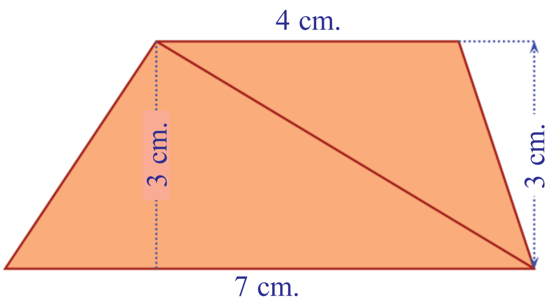


So, the area of this triangle is

$$\frac{1}{2} \times 7 \times 3 = 10\,\frac{1}{2} \text{ sq.cm.}$$

What about the area of the upper triangle?

For that, we need to measure the length of the top side and the distance from the opposite vertex. Since the top and bottom sides are parallel, this distance is 3 centimetres again.

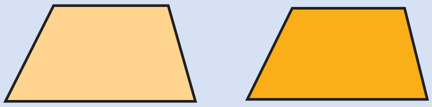


Area of the upper triangle is

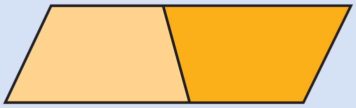
$$\frac{1}{2} \times 4 \times 3 = 6 \text{ sq.cm.}$$

Another Way

Cut out two equal trapeziums.



Flip one upside down and join with the other as shown below:

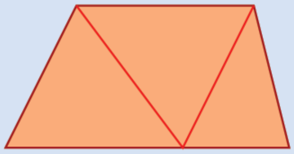


Now we have a parallelogram (Why?) Its top and bottom sides are the parallel sides of the trapezium joined together; and is height is that of the trapezium.

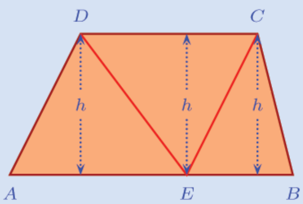
So, the area of the parallelogram is product of the sum of the parallel sides of the trapezium and its height. The area of a trapezium is half this product.

Trapezium and triangles

See this picture:



A trapezium is cut into three triangles. The area of the trapezium is the sum of the areas of the triangles.



All these triangles have the same height. So the area of the trapezium is

$$\left(\frac{1}{2}\times h\times AE\right)+\left(\frac{1}{2}\times h\times EB\right)+\left(\frac{1}{2}\times h\times CD\right)$$

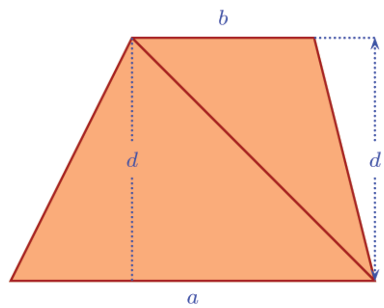
$$= \frac{1}{2}\times h\left(AE+EB+CD\right)$$

$$= \frac{1}{2}\times h\left(AB+CD\right)$$

The area of the trapezium is the sum of the areas of these triangles; that is $16\frac{1}{2}$ square centimetres.

What all lengths did we use to compute this area?

To understand the general method, let's take the length of the parallel sides of a trapezium as a, b and the distance between them as d.



In the figure, the area of the lower triangle is $\frac{1}{2}ad$ and the area of the upper triangles to $\frac{1}{2}bd$. So the area of the trapezium.

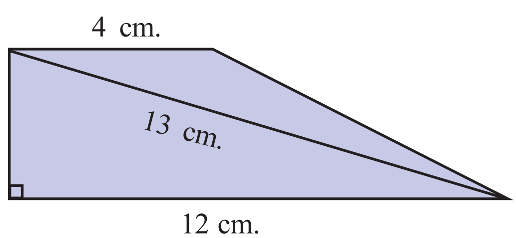
$$\frac{1}{2}ad + \frac{1}{2}bd = \frac{1}{2}(a+b)d$$

How about saying this in usual language, without any algebra?

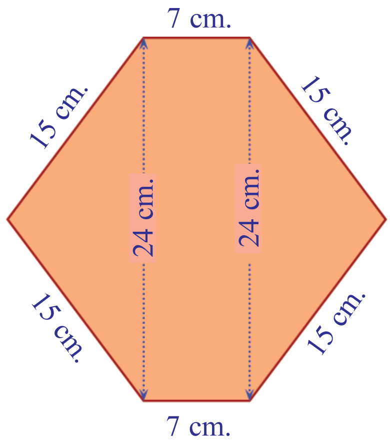
The area of a trapezium is half the product of the parallel sides and the distance between them.

1) The lengths of the parallel sides of a trapezium are 30 centimetres, 10 centimetres and the distance between them is 20 centimetres. What is its area?

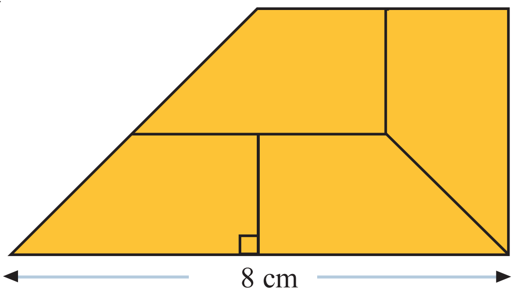
2) Compute the area of the trapezium shown below:



3) Compute the area of the hexagon below.



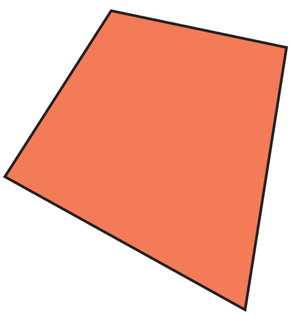
4) This is a picture drawn in the lesson **Construction of Quadrilaterals.**



What is the area of the large trapezium made up of the four smaller ones?

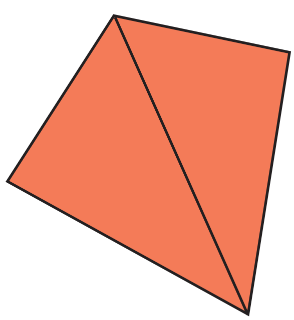
Quadrilateral

How do we calculate the area of the quadrilateral below?

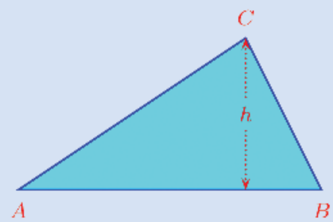


How about drawing a diagonal and split it into two triangles?

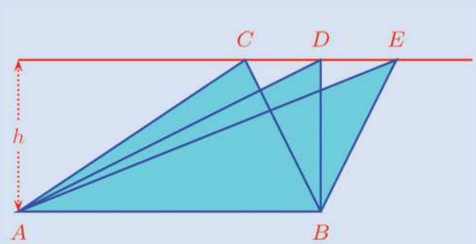
If we know the length of this diagonal, what more do we need to calculate the areas of these triangles?



Unchanging area and changing perimeter

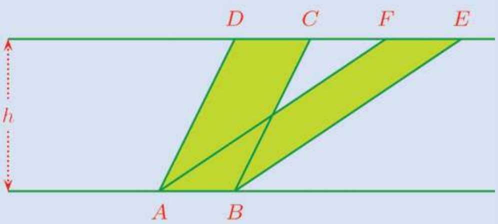


The area of ΔABC above is $\frac{1}{2} \times AB \times h$. If we move C along a line parallel to AB, the triangle changes.



In each of ΔABC, ΔABD, ΔABE the length of the perpendicular to AB from the third vertex is h, and so these triangles have the same area. But we can see that the perimeters are different. What is the speciality of the triangle of least perimeter?

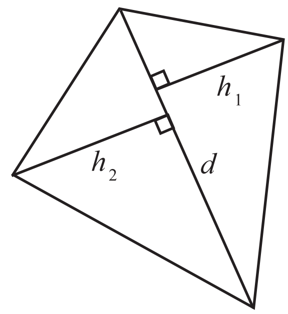
Minimum Perimeter



The area of the parallelogram $ABCD$ in the picture above is $AB \times h$. Even if we shift the side CD to EF parallel to AB, the area remains $AB \times h$. We can draft CD to any position on the upperline, without changing the area. But the perimeter changes. What is the speciality of the parallelogram of least perimeter? Like this, can we change the perimeter of a trapezium without changing the area?

What is the speciality of the trapezium of least perimeter?

We need only know the distance from opposite vertices to this diagonal.



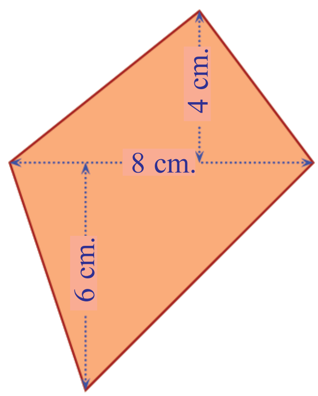
Taking the length of the diagonal as d and the perpendicular distances as h_1 and h_2, the area of the quadrilateral is

$$\frac{1}{2}dh_1 + \frac{1}{2}dh_2 = \frac{1}{2}d\,(h_1 + h_2)$$

In usual language, we can say it like this.

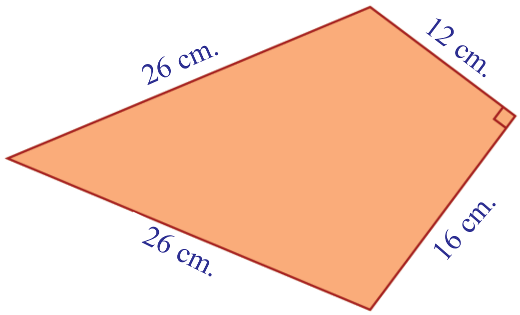
The area of a quadrilateral is half the product of a diagonal and the sum of the perpendicular distances from the opposite vertices to this diagonal.

1) What is the area of the quadrilateral shown below?

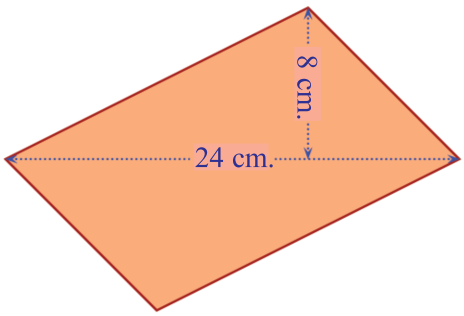


2) Prove that for any quadrilateral with diagonals perpendicular, the area is half the product of the diagonals.

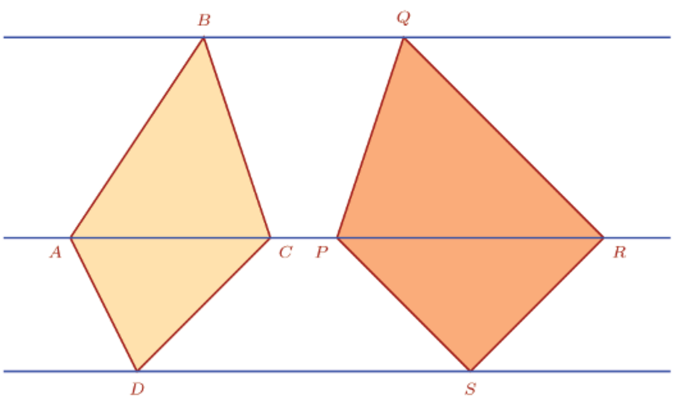
3) Compute the area of the quadrilateral shown below:



4) Compute the area of the parallelogram shown below:



5) The three blue lines in the picture below are parallel:



Prove that the areas of the quadrilaterals *ABCD* and *PQRS* are in the ratio of the lengths of the diagonals *AC* and *PR*.

i) How should the diagonals be related for the quadrilaterals to have equal area?

ii) Draw two quadrilaterals, neither parallelograms nor trapeziums, of area 15 square centimetres.

Looking back

Learning outcomes	What I can	With teacher's help	Must Improve
• Describing methods of drawing parallelograms of area equal to that of a rectangle.			
• Understanding the methods of computing the areas of a parallelogram			
• Understanding the method of computing the area of a rhombus in terms of its diagonals.			
• Drawing rhombuses of specified areas.			
• Describing the method of drawing isosceles trapezium, of area equal to that of a rectangle.			
• Understanding the general method of computing the area of any quadrilateral.			

9

Negative Numbers

+	-5	-4	-3	-2	-1	0	1	2	3	4	5
5	0	1	2	3	4	5	6	7	8	9	10
4	-1	0	1	2	3	4	5	6	7	8	9
3	-2	-1	0	1	2	3	4	5	6	7	8
2	-3	-2	-1	0	1	2	3	4	5	6	7
1	-4	-3	-2	-1	0	1	2	3	4	5	6
0	-5	-4	-3	-2	-1	0	1	2	3	4	5
-1	-6	-5	-4	-3	-2	-1	0	1	2	3	4
-2	-7	-6	-5	-4	-3	-2	-1	0	1	2	3
-3	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2
-4	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1
-5	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0

×	-5	-4	-3	-2	-1	0	1	2	3	4	5
5	-25	-20	-15	-10	-5	0	5	10	15	20	25
4	-20	-16	-12	-8	-4	0	4	8	12	16	20
3	-15	-12	-9	-6	-3	0	3	6	9	12	15
2	-10	-8	-6	-4	-2	0	2	4	6	8	10
1	-5	-4	-3	-2	-1	0	1	2	3	4	5
0	0	0	0	0	0	0	0	0	0	0	0
-1	5	4	3	2	1	0	-1	-2	-3	-4	-5
-2	10	8	6	4	2	0	-2	-4	-6	-8	-10
-3	15	12	9	6	3	0	-3	-6	-9	-12	-15
-4	20	16	12	8	4	0	-4	-8	-12	-16	-20
-5	25	20	15	10	5	0	-5	-10	-15	-20	-25

Old Sums

We have seen in class 7, how temperatures below zero are denoted using negative numbers. The temperature at which water freezes to ice, is taken as zero degree Celsius, written 0^0 C. To denote temperatures below this, we have to write $-1°C$, $-20.5°C$ and so on.

Measurements and numbers

Men invented numbers to denote the results of measurements. During the pastoral stage, men needed only natural numbers, to count ones' friends and cattle. Once they started agriculture, men need to measure, length, weight, time and so on. Which need units of measurement? For example, we now use metre to measure length, kilograms to measure weight and seconds to measure time. Fractions were invented to denote quantities smaller than the unit.

We have also seen how negative numbers are used to denote points in some games and scores in some tests. We also did some computations based on these.

For example, we write

$$3 - 7 = -(7 - 3) = -4$$

$$2 - 5\frac{1}{2} = -\left(5\frac{1}{2} - 2\right) = -3\frac{1}{2}$$

We also stated the general principle of these in Class 7.

> For any two positive numbers, subtracting the larger from the smaller means, taking the negative of the smaller subtracted from the larger.

In algebra we write it like this:

For any two positive numbers x, y with $x < y$,

$$x - y = -(y - x)$$

We also saw calculations like

$$-3 + 7 = 7 - 3 = 4$$

$$-2 + 5\frac{1}{2} = 5\frac{1}{2} - 2 = 3\frac{1}{2}$$

The general principle of these:

> For any two positive numbers, adding the second to the negative of the first means, subtracting the first from the second.

In algebra,

for any two positive numbers x, y

$-x + y = y - x$

Using these two together, we can do calculations like

$$-7 + 3 = 3 - 7 = -4$$

$$-5\frac{1}{2} + 2 = 2 - 5\frac{1}{2} = -3\frac{1}{2}$$

Moreover, we have done computations such as

$$-3 - 7 = -(3 + 7) = -10$$

$$-2 - 5\frac{1}{2} = -\left(2 + 5\frac{1}{2}\right) = -7\frac{1}{2}$$

And the general rule for these:

For any two positive numbers, subtracting the second from the negative of the first means finding the negative of their sum.

In algebra,

for any two positive numbers x, y

$-x - y = -(x + y)$.

Using the principles above, compute the following:

i) $5 - 10$ ii) $-10 + 5$

iii) $-5 - 10$ iv) $-5 - 5$

v) $-5 + 5$ vi) $-\frac{1}{2} + 1\frac{1}{2}$

vii) $-\frac{1}{2} - 1\frac{1}{2}$ viii) $-\frac{1}{2} + \frac{1}{2}$

Operations on natural numbers

The operation of adding two natural numbers arise from counting the total of two groups. In counting things of the same kind, it is often convenient to group them first. The idea of repeated addition and naming it as multiplication arises from this recognition. For example, in counting coconuts and other things, people often count them in twos and threes and then multiply by two or three.

Negative Speed

There are some conveniences in Physics also, in using negative numbers. Let's take a look again at such an example seen in class 7. (The sections **speed math** and **Negative speeds** of the lesson, **Negative Numbers**)

It is a common experience that things thrown up rise for some time and then fall back to the ground. There is some math behind this. If we throw something straight up, during every second of the flight up it loses speed by 9.8 metre/second decreasing; thus, when there is no speed at all, it begins to fall down. During every second of this fall, speed increases 9.8 metre/second.

Suppose we throw something straight up with a speed of 49 metre/second. After one second, the speed becomes $49 - 9.8 = 39.2$ metre/second; after two seconds, $49 - (2 \times 9.8) = 29.4$ metre/ second.

At 5 seconds, speed becomes $49 - (5 \times 9.8) = 0$. Then the downward journey starts with speed increasing at the old rate.

Operations on fractions

The need to find the total of two lengths or weights less than the unit of measurement, led to the idea of addition of fractions. Taking a part of the unit and then a part of this part, leads to multiplication of fractions. It is not repeated addition as in the case of natural numbers. Thus in mathematics, operations of the same name (and written using the same symbol) may differ according to context.

So 7 seconds after throwing, what would be the speed?

At 5 seconds, the speed is zero. We want the speed 2 seconds after this. During these 2 seconds, the journey is downward with increasing speed.

Speed after 2 seconds is $-2 \times 4.0 = 9.8$ metre/second.

What would be the speed after 9 seconds?

Let's put the entire travelogue in algebra.

What would be the speed, t seconds after throwing?

Till 5 seconds, the journey is upward with decreasing speed.

Thus if $t < 5$, the speed is $49 - 9.8t$ metre/second.

At 5 seconds speed to zero; for every second after it, downward journey with increasing speed. That is, if $t > 5$, the journey to downward for $t - 5$ seconds.

So, speed is $9.8\,(t - 5) = 9.8t - 49$ metre/second.

So if we write the speed after t seconds as v, we have to split the relation between v and t into different cases like this :

$$v = \begin{cases} 49 - 9.8t, & \text{if } t < 5 \\ 0, & \text{if } t = 5 \\ 9.8t - 49, & \text{if } t > 5 \end{cases}$$

How about writing downward speeds as negative numbers?

For example, to find the speed after 8 seconds, we have to use the third part of the above equation. From that we get the speed as $(9.8 \times 8) - 49 = 29.4$ metre/second.

Since this speed is downwards, we write it as -29.4 metre/second.

Now if we take $t = 8$ in the first part $49 - 9.8t$, then also we get $v = 49 - (9.8 \times 8) = -29.4$ metre/second.

In general, if we use negative numbers for downward speeds like this, then we can condense the relation between time and speed into the simple equation

$$v = 49 - 9.8t$$

There is another convenience in this – we get to know whether the journey is upward or downward by noting whether it is positive or negative.

Math World

From the very old days, astronomers used many mathematical computation to find out orbits of planets and other things. But the idea of using mathematics to describe general principles of motion and energy developed only in 14th century Europe. Continuing this tradition, in the 17th century, Galileo Gaelei of Italy discovered that for an object falling downwards, the distance travelled is equal to the square of the time travelled multiplied by a specific number.

He stated the relation between mathematics and physics like this:

Philosophy is written in the grand book of the universe. To understand it, we must know the language in which it is written. It is written in the language of mathematics.

An object is thrown straight up with a speed of 98 metre/second. What is the single equation to find the speed at every second? After how many seconds does it reach the highest point? What is the speed after 13 seconds? Is the journey upward or downward at this time?

New sums and differences

In the equation $v = 49 - 9.8t$,

when we take $t = 3$, we get $v = 19.6$,

when we take $t = 5$, we get $v = 0$,

when we take $t = 7$, we get $v = -19.6$.

Here, as we take different numbers as t, we get positive numbers, zero or negative numbers as v.

All kinds of numbers are denoted by the letter v.

This is a convention in algebra. Positive and negative numbers are denoted by letters, without any sign. Thus letters such as x, y and so on are taken as positive or negative numbers, according to context.

Now look at the equation

$$z = x + y$$

If we take $x = -10, y = 3$ in this, we get

$$z = -10 + 3 = -7$$

Like this,

if we take $x = -3, y = 10$, we get

$$z = -3 + 10 = 7$$

if we take $x = 10, y = -3$, we get

$$z = 10 + (-3)$$

What does this mean?

In adding two positive numbers, we can take either one first. If this is to hold here also, we must take $10 + (-3)$ to mean $-3 + 10$.

Thus,

$$z = 10 + (-3) = -3 + 10 = 10 - 3 = 7$$

Like this, take $x = 8, y = -2$ and compute z

What if we take $x = -10,\ y = -3$

$$z = -10 + (-3)$$

As before, if we take adding -3 as subtracting 3, then

$$z = -10 + (-3) = -10 - 3 = -13.$$

What if we take $x = -5$ and $y = -6$.

Like this, we compute

$$7 + (-5) = 7 - 5 = 2$$

$$-7 + (-5) = -7 - 5 = -12$$

In general we can state this as follows:

Adding the negative of a positive number means subtracting that positive number.

We must also explain the meaning of subtraction. For example, look at the equation.

$$z = x - y$$

If we take $x = 10, y = 3$ in this

$$z = 10 - 3 = 7$$

If we take $x = 3, y = 10$, then

$$z = 3 - 10 = -7$$

What if we take $x = 10, y = -3$?

$$z = 10 - (-3)$$

We have not seen any instance of subtracting a negative number.

What does it mean?

We can think like this: the meaning of $10 - 3$ is, what must be added to 3 to get 7, right? In other words, $8 + 7 = 10$; and so $10 - 3 = 7$.

According to this, $10 - (-3)$ means, finding what number must be added to -3 to get 10.

3 added to -3 gives 0. To get 10, we must add 10 again. We must add $10 + 3 = 11$ in all. In short,

$$10 - (-3) = 10 + 3 = 13$$

Thus the meaning we give to subtracting -3 from 10 is adding 3 to 10.

So what if we take $x = -10, y = -3$?

$$z = -10 - (-3)$$

Here also, if we take subtracting -3 to mean adding 3, we get

$$z = -10 + 3 = -7$$

According to this meaning,

$$7 - (-5) = 7 + 5 = 12$$

$$15 - (-8) = 15 + 8 = 23$$

$$-7 - (-5) = -7 + 5 = -2$$

$$-15 - (-8) = -15 + 8 = -7$$

and so on.

In general, we have the following:

Subtracting the negative of a positive number means adding that positive number.

Definitions

A definition is an explanation of the meaning of a word or idea. For example,

An insect is a six-legged creature is a definition in biology.

Likewise.

$\frac{1}{2} \times \frac{1}{3}$ means $\frac{1}{2}$ of $\frac{1}{3}$ is a mathematical definition. We compute $\frac{1}{2} \times \frac{1}{3}$ as $\frac{1}{6}$ on the basis of this definition.

According to this definition.

$$0 - (-3) = 0 + 3 = 3$$

Just as wrote $0 - 3$ as -3, we may write $0 - (-3)$ as $-(-3)$. That is,

$$-(-3) = 0 - (-3) = 0 + 3 = 3$$

What about $-(-(-3))$?

$$-(-3) = 3;\ \text{so}\ -(-(-3)) = -3$$

In short, we can say this :

The negative of the negative of a number is that number itself.

That is,

$$-(-x) = x, \text{ for any number } x.$$

1) Take as x different positive numbers, negative numbers and zero, and compute $x + 1$, $x - 1$, $1 - x$. Check whether the equations below hold for all numbers.

i) $(1 + x) + (1 - x) = 2$ ii) $x - (x - 1) = 1$

iii) $1 - x = -(x - 1)$

2) Taking different numbers as x, y and compute $x + y$, $x - y$. Check whether the following hold for all kinds of numbers.

i) $(x + y) - x = y$

ii) $(x + y) - y = x$

iii) $(x - y) + y = x$

Applications

Imagine starting at a point, travelling some distance in one direction and then some more distance along the same or opposite direction. We want to find out the final position with respect to the starting point. Given below is a table showing some examples of this in different cases.

First trip	Second trip	Final position
5 metres right	3 metres right	8 metres right
3 metres right	5 metres right	
5 metres right	3 metres left	2 metres right
3 metres left	5 metres right	
5 metres left	3 metres right	
3 metres right	5 metres left	
5 metres left	3 metres left	
3 metres left	5 metres left	

To avoid writing "right" or "left", let's write distances to the right as positive numbers and distances to the left as negative numbers.

First trip	Second trip	Final position
5 metres	3 metres	8 metres
3 metres	5 metres	8 metres
5 metres	–3 metres	2 metres
–3 metres	5 metres	2 metres
–5 metres	3 metres	–2 metres
3 metres	–5 metres	–2 metres
–5 metres	–3 metres	–8 metres
–3 metres	–5 metres	–8 metres

In each row of this table, isn't the last number the sum of the first two?

So, if we write distances as positive or negative numbers, then to find the final position, we need only add the distances. For example, travelling 23 metres to the left first and then travelling 15 metres to the right gives the final position as

$$-23 + 15 = -8$$

That is 8 metres to the left of the starting point.

In general, if the first trip is x metres and the second y metres then the final position can be found using the single equation

$$z = x + y$$

Just think how many different equations we would need if instead of using positive and negative numbers, we specify distances as right or left.

Different distances

What if do the problem of finding the final position on travelling first in one direction and then in the same or opposite direction, without using negative numbers? Let's take the distance travelled first as x, the distance travelled next as y and the final position as z. If z and y are in the same direction, we can using $z = x + y$.

If x is to the right and y to the left?

If $x > y$, then $z = x - y$ to the right and if x < y $z = y - x$ to the left what if x is to the left and y to the right.

There are other conveniences in denoting positive and negative numbers in the same way, using letters. For example, look at a general principle seen earlier.

> For any two positive numbers, subtracting the larger from the smaller means, taking the negative of the smaller subtracted from the larger.

For any two positive numbers x, y with $x < y$,

$$x - y = -(y - x)$$

What if x is not less than y?

For example, taking $x = 7, y = 3$ gives.

$$x - y = 7 - 3 = 4$$

$$y - x = 3 - 7 = -4$$

$$-(y - x) = -(-4) = 4$$

Thus we have $x - y = -(y - x)$.

Take other such pairs of numbers as x, y and check.

Don't we get $x - y = -(y - x)$ for all these?

Again should we take x, y as positive numbers only? For example $x = 8, y = -3$ gives

$$x - y = 8 - (-3) = 11$$

$$y - x = -3 - 8 = -11$$

$$-(y - x) = -(-11) = 11$$

Thus the equation $x - y = -(y - x)$ is true here also.

Take other pairs of positive or negative numbers and check. Doesn't this equation hold? So this principle is true for all pairs of numbers.

For any two numbers, subtracting one from another is equal to the negative of subtracting in reverse.

$x - y = -(y - x)$ for all numbers x, y.

Now let's look at the second general principle:

For any two positive numbers, adding the second to the negative of the first means, subtracting the first from the second.

That is,

For any two positive numbers x, y

$$-x + y = y - x.$$

Let's check whether this holds for all numbers positive, negative or zero.

For example, taking $x = 7$, $y = 3$, we get

$$-x + y = -(-7) + 3 = 10$$

$$y - x = 3 - (-7) = 3 + 7 = 10$$

So that we find

$$-x + y = y - x$$

How about $x = -8$, $y = -5$?

$$-x + y = -(-8) + (-5) = 8 + (-5)$$

$$= 8 - 5 = 3$$

$$y - x = -5 - (-8) = -5 + 8$$

$$= 8 - 5 = 3$$

Again, we find

$$-x + y = y - x$$

Check for other pairs of numbers.

We can see that the equation is true for all numbers.

So, our general principle can be extended like this.

For any two numbers, adding the second to the negative of the first is equal to subtracting the first from this second.

$-x + y = y - x$ for all numbers x, y.

What is the third principle we noted earlier?

For any two positive numbers, subtracting the second from the negative of the first means finding the negative of their sum.

What is its algebraic form?

Check whether this equation holds for all kinds of numbers.

1) Check whether the equations are identities. Write the patterns got from each, on taking $x = 1, 2, 3, 4, 5$ and $x = -1, -2, -3, -4, -5$.

i) $-x + (x + 1) = 1$ ii) $-x + (x + 1) + (x + 2) - (x + 3) = 0$

iii) $-x - (x + 1) + (x + 2) + (x + 3) = 4$

2) Taking different numbers, positive, negative and zero, as x, y, z and compute $x + (y + z)$ and $(x + y) + 3$. Check whether the equation, $x + (y + z) = (x + y) + z$ holds for all these numbers.

New Multiplication

Let's again think about a point moving along a straight line. This time, we consider the speed also. If the speed is the same throughout the journey, then to find the distance from the starting point at any time, we need only multiply the speed by the time. For example, if the speed is 10 metre/second, then in 3 seconds, it would be 30 metres away.

The journey can be either to the right or to the left of the starting point. As before, we take distances to the right as positive and distances to the left as negative.

Let's take the speed as 10 metre/second. If the distance travelled in t seconds is taken as s, what is the relation between s and t?

If the journey is to the right, then $s = 10t$ metres and if it is to the left, then $s = -10t$ metres. Thus we have to use two equations.

In general, if the speed is v metre/second, then the distance travelled towards right is $s = vt$ metres and towards left is $s = -vt$ metres.

If we take speed to the right as positive and to the left as negative, can we write.

$$s = vt$$

for both?

For example, suppose the journey is to the left. In 2 seconds, the position reached is 20 metres to the left.

According to our conventions, we take speed as –10 metre/second and the position reached as –20 metres away. So if the equation $s = 10t$ is to hold, we must take.

$$(-10) \times 2 = -20$$

We give meaning to similar products like this:

$$(-5) \times 8 = -40$$
$$(-1) \times 1 = -1$$
$$-\frac{1}{2} \times 4 = -2$$
$$\left(-\frac{1}{2}\right) \times \frac{1}{3} = -\frac{1}{6}$$

Then another question arises: what is the meaning of $5 \times (-8)$.

In multiplying two positive numbers, taking them in any order gives the same product, right? For example, $5 \times 8 = 8 \times 5 = 40$.

For this to be true for negative numbers also, we must take

$5 \times (-8) = (-8) \times 5$.

Thus we take

$$5 \times (-8) = (-8) \times 5 = -40$$
$$1 \times (-1) = (-1) \times 1 = -1$$
$$\frac{1}{2} \times \left(-\frac{1}{3}\right) = \left(-\frac{1}{3}\right) \times \frac{1}{2} = -\frac{1}{6}$$

So,

$$3 \times (-5) = -(3 \times 5) = -15$$
$$(-3) \times 5 = -(3 \times 5) = -15.$$

The general definition is as follows:

The product of a positive number and the negative of a positive number means, the negative of the product of these positive numbers.

For any two positive numbers x, y

$$(-x)\ y = x\ (-y) = -(xy)$$

Let's change our time and distance example slightly. Suppose we watch a point travelling with constant speed along a straight line, only from a certain stage of its journey. Let's take the position at the time of observation as 0. Suppose that the journey is from left to right at 10 metre/second. In 2 seconds from the time of observation, the point is 20 metres to the right of 0. What about the position, 2 seconds before the start of observation?

Now what if the journey is from right to left?

Where is the position, 2 seconds after starting observation?
And 2 seconds before?

Speed	Time	Distance
10 metre/second to the right	2 seconds after	20 metres right
10 metre/second to the right	2 seconds before	20 metres left
10 metre/second left	2 seconds after	20 metres left
10 metre/second left	2 seconds before	20 metres right

Let's write speed and distance to the right as positive numbers and those to the left as negative numbers.

Speed	Time	Distance
10 metre/second	2 seconds after	20 metre
10 metre/second	2 seconds before	–20 metre
–10 metre/second	2 seconds after	–20 metre
–10 metre/second	2 seconds before	20 metre

In the case of time also, we can do away with the adjectives after and before, by taking times after the start of observation as positive numbers and those before as negative numbers.

v (metre/second)	t (second)	s (metre)
10	2	20
10	– 2	– 20
– 10	2	– 20
–10	– 2	20

In this situation also, does the single equation

$$s = vt$$

give the relation between time, speed and distance in all cases?

According to our definition of the product of positive numbers and negative numbers, this equation is correct for the first three rows of the table. What about the last line?

$v = -10,\ t = -2$ gives,

$$vt = (-10) \times (-2)$$

Negative Multiplication

The idea of the products involving negative numbers was first considered by *Brahmagupta* of India; in the seventh century AD. It is described in his book Brahmasphutiyasiddhanta

He makes definition such as negative times negative is positive to give a single general method for formulizing problems involving a number and its square, and for finding their solutions.

But we have not said so far, what it means to multiply a negative number by a negative number.

Here, we have $s = 20$. So if we want the equation $s = vt$ to hold, we must define.

$$(-10) \times (-2) = 20$$

Similarly we define

$$(-3) \times (-4) = 12$$
$$(-5) \times (-8) = 40$$
$$\left(-\frac{1}{3}\right) \times \left(-\frac{1}{2}\right) = \frac{1}{6}$$

The general definition is this:

The product of the negatives of two positive numbers means the product of these positive numbers.

For any two positive numbers *x, y*

$$(-x)\ (-y) = xy$$

1) Take various positive and negative numbers as x, y, z and compute $(x + y)\,z$ and $xz + yz$. Check whether the equation $(x + y)\,z = xz + yz$ holds for all these.

2) In each of the following equations, take x as the given numbers and compute the numbers y.

   i) $y = x^2,\ x = -5,\ x = 5$ ii) $y = x^2 + 3x + 2,\ x = -2$

   iii) $y = x^2 + 5x + 4,\ x = -2,\ x = -3$

   iv) $y = x^3 + 1,\ \ x = -1$

   v) $y = x^3 + x^2 + x + 1,\ x = -1$

3) For a point starting at a point P and travelling along a straight line, time of travel is taken as t and the distance from P as s. The relation between s and t is found to be $s = 12t - 2t^2$, where distances to the right are taken as positive numbers and to the left as negative numbers.

   i) Is the position of the point to the right or left of P, till 6 seconds?

ii) Where is the position at 6 seconds?

iii) After 6 seconds?

(Here it is convenient to write $12t - 2t^2 = 2t(6 - t)$).

4) Natural numbers, their negatives and zero are together called integers. How many pairs of integers are there, satisfying the equation, $x^2 + y^2 = 25$?

Negative division

For positive numbers, division is defined in terms of multiplication, isn't it? For example $6 \div 2$ denotes the operation of finding out which number 2 is to be multiplied by to get 6. In other words, $6 \div 2 = 3$ because $2 \times 3 = 6$.

Similarly, we say $1\frac{1}{2} \div \frac{3}{4} = 2$, because $2 \times \frac{3}{4} = 1\frac{1}{2}$ (the section **Fraction division** of the lesson **Part and Times** in class 6).

So, $(-6) \div 2$ means finding out the number 2 should be multiplied by to get -6. 2 multiplied by -3 gives -6, right?

So, we have $(-6) \div 2 = -3$.

What about dividing -15 by 3 ?

And $6 \div (-2)$?

By which number should we multiply -2 to get 6?

So, $6 \div (-2) = -3$.

What if $20 \div (-5)$?

Can you calculate $(-6) \div (-2)$.

In algebra, we usually write $x \div y$ as $\frac{x}{y}$.

So, in the equation $z = \frac{x}{y}$

$x = -6, y = 2$ gives $z = -3$.

$x = 6, y = -2$ gives $z = -3$.

$x = -6, y = -2$ gives $z = 3$.

Power of –1

$$(-1)^2 = (-1) \times (-1) = 1$$
$$(-1)^3 = (-1)^2 \times (-1) = 1 \times (-1) = -1$$
$$(-1)^4 = (-1)^3 \times (-1) = (-1) \times (-1) = 1$$
$$(-1)^5 = (-1)^4 \times (-1) = 1 \times (-1) = -1$$

What do you see? Find some more powers. Dont' you get even powers as 1 and add powers as -1?

In general, we have for any natural number n,

$$(-1)^n = \begin{cases} 1, & \text{if } n \text{ is even} \\ -1, & \text{if } n \text{ is even} \end{cases}$$

Square root

What is the square root of 25 ?

$$5 \times 5 = 25$$

So, 5 is a square root of 25.

We have now seen that

$$(-5) \times (-5) = 25$$

also. Thus -5 is also a square root of 25.

Similarly, any non-zero number has two square roots, one positive and the other negative.

It is the positive square root that we denote by the $\sqrt{\ }$ sign.

For example, $\sqrt{25} = 5$.

The second square root -5 is thus $-\sqrt{25}$.

1) In the equation $y = \frac{1}{x}$ take x as the numbers $-\frac{2}{3}, -\frac{1}{2}, -\frac{3}{5}$ and compute y.

2) In the equation $y = \frac{1}{x-1} + \frac{1}{x+1}$, take $x = -2$ and $x = -\frac{1}{2}$ and compute y.

3) In the equation $z = \frac{x}{y} - \frac{y}{x}$, take x as the numbers given below and calculate the number z.

i. $x = 10, y = -5$ ii. $x = -10, y = 5$

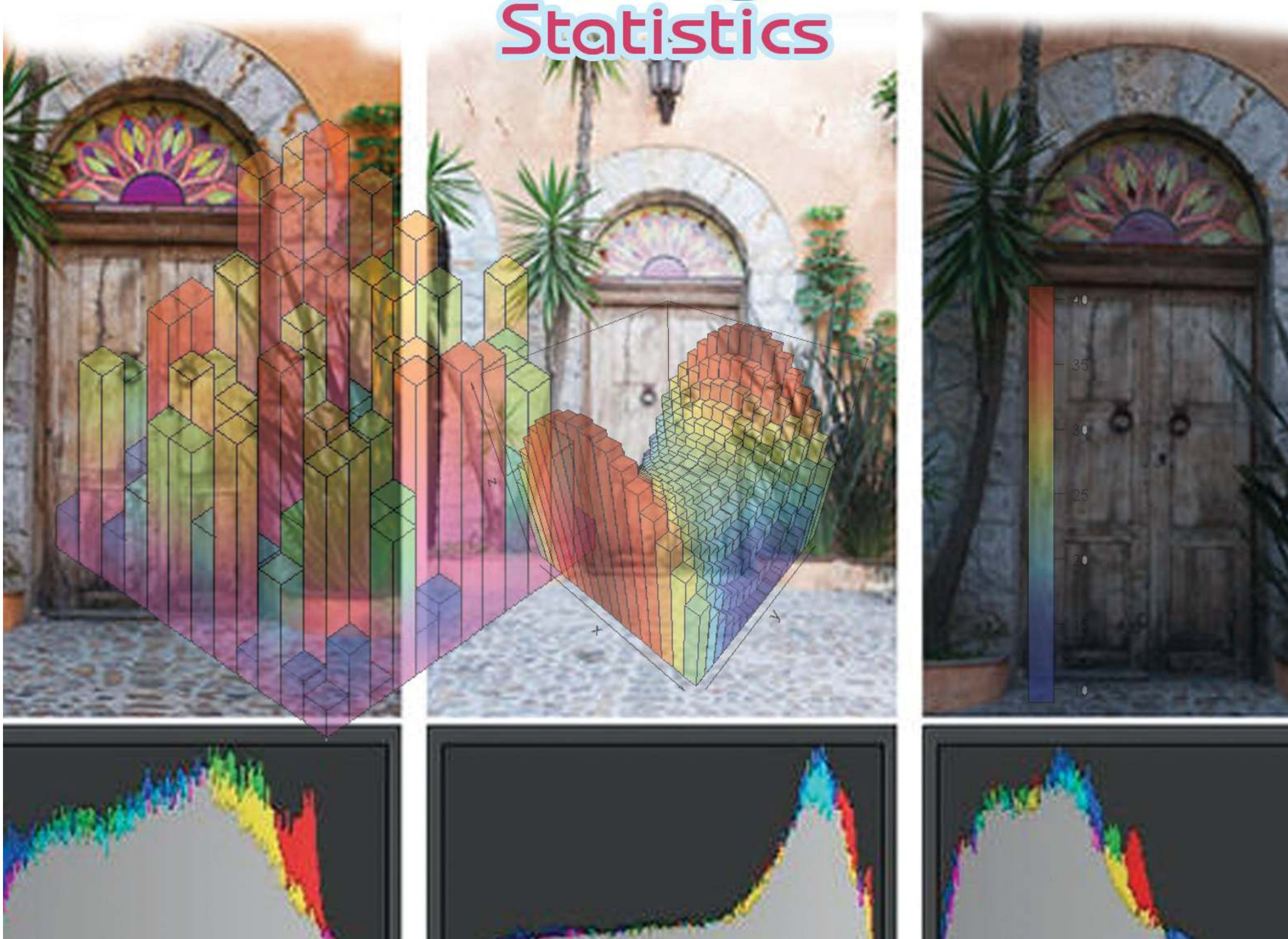
iii. $x = -10, y = -5$ iv. $x = 5, y = -10$

v. $x = -5, y = 10$

Looking back

Learning outcomes	What I can	With teacher's help	Must Improve
• Understanding the convention in algebra of denoting positive and negative numbers by letters without sign, and its convenience.			
• Realising that when positive and negative numbers are taken together, new definitions of addition and subtraction are needed and to understand this definition.			
• Realising the need for defining multiplication in certain contexts involving negative numbers and to understand this definition.			
• Understanding division as the inverse of multiplication for negative numbers, as in the case of positive numbers.			
• Simplifying algebraic expression taking positive and negative numbers as the letters.			

10
Statistics



Tabulation

There are 40 children in class 8A of the school. In a Health Club programme, their blood groups were identified and listed as below:

O+	B+	O+	AB+	AB−	B−
O+	AB−	AB+	AB+	B−	AB+
A+	O+	O+	O+	O+	A+
O−	A+	A+	O+	O+	O+
B+	B+	A+	A+	B+	O+
AB+	A+	B+	B+	O+	A+
B−	O+	O+	B+		

i) How many children have O− blood type?

ii) How many have B−?

iii) How many have O+?

iv) Which blood type do the most number of children have?

v) Which type do the least number of children have?

To answer the first question, we need only count the O− types only for the second, we count the B− types and for the third, the O+ types.

What about the fourth question?

We have to add each type separately, right?

Here, it is convenient to record this counting first:

Blood Group	Number
A+	8
B+	7
AB+	5
O+	14
B−	3
AB−	2
O−	1

Now can't you answer the last two questions easily?

Another problem.

The scores children in a class got in a test are listed below:

8	7	6	3	8	8	7	7	6
7	9	7	6	8	7	2	6	7
10	6	7	3	9	5	4	5	4
4	4	5	8	10	8	8	9	7
7	6	8	8	7	4	5	9	8

i) What is the score got by the most number of children?

ii) How many children got 8 or more?

iii) How many got less than 8?

iv) How many children got 10?

Let's make a table as before.

We must note how many times each score occurs.

The lowest score is 2 and the highest is 10.

Write the numbers from 2 to 10 in a column and check how many times each is repeated. We can use the method of tallies seen in class 5.

Score	Tally	Numbers of children
2	|	1
3	||	2
4	𝍸	5
5	||||	4
6	𝍸 |	6
7	𝍸 𝍸 |	11
8	𝍸 𝍸	10
9	||||	4
10	||	2
	Total	45

Now it is easy to answer all questions above, just by looking at the table, isn't it?

The table shows how many times each score occurs, such as 2 once, 3 twice, 11 times so on. In tables of this kind, the number of occurrences is generally called frequency and the table itself is called a frequency table.

1) The number of members in 50 households of a village are listed below.

8	6	9	4	4	2	6	5	4	3
7	3	3	2	3	7	6	3	2	5
5	13	9	9	7	4	4	5	4	3
3	7	2	3	3	10	8	6	6	4
2	4	5	4	3	8	7	5	6	3

Make a frequency table and answer these questions:

i) How many households have just two members?

ii) How many households have four or less?

iii) How many households have ten or more?

iv) Households of what size occurs the most?

2) There are 44 children in class 8B. The list shows how far they come from, in kilometres.

6	2	7	12	1	9	2	6
5	7	3	4	1	5	4	4
5	8	6	5	2	5	9	5
11	12	1	9	2	14	4	7
9	6	6	7	3	2	6	3
4	7	9	3				

Make a frequency table and answer these questions:

i) How many children are from exactly 1 kilometre away?

ii) How many are from more than 5 kilometres?

iii) How many are from between 5 and 10 kilometres?

iv) How many are from more than 10 kilometres?

3) The scores of 35 children in a test are given below:

15	10	18	11	19	16	15	17	14	18	13	15
17	16	15	14	15	17	14	15	13	16	11	11
16	20	13	12	10	16	17	13	12	14	12	

Make a frequency table and answer these questions:

i) How many children scored 20?

ii) How many children got scores between 10 and 20?

iii) How many scored less than 10?

iv) What is the score most number of children got?

Another form

The runs that a batsman got in 50 one-day cricket matches are given below.

50	0	49	60	100	68	27	48	15	65	101	45	2
52	25	18	29	53	72	90	32	81	28	104	35	49
2	60	87	71	38	102	35	71	68	20	10	30	55
47	21	35	12	20	11	27	43	38	40	48		

i) How many centuries did he get ?

ii) How many half-centuries?

iii) In how many games did he score less than 50?

Here the lowest score is zero and the highest is 104.

To make a table as we did so far, we would have to write all numbers from 0 to 104. But all such numbers are not really needed. Moreover from such table, we don't get a general idea of the player's performance.

So, we do it in a slightly different way.

Instead of writing the actual runs in a column, we group them into classes as centuries (100 or more), half-centuries (50 - 99) and less than a half century (less than 50) and make a table:

Class	Tally	Number of games
0 - 49	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	31
50 - 99	𝍸 𝍸 𝍸	15
100 and more	||||	4

Now from the table, can't we easily answer the questions asked?

Suppose we want to analyse the performance in little more detail, to answer questions like these:

- In how many games did he score less than 10?
- In how many games did he score between 90 and 100?
- Between 40 and 50?

Then we would have to group the score into suitable classes and make a table.

We can group the scores as 0 to 9, 10 to 19, 20 to 29 and so on and count the number of games in each.

On tables

To draw conclusions from a collection of data, we have to first put them in order. One method of such an arrangement is to classify them and form a table. A type of table used in statistics is the frequency table.

When we tabulate data like this, some information is lost. For example, when the entire data collected on incomes is presented as income groups and the number of people belonging to each group, we cannot find the actual income of each person from it.

But from such a table, we can get a general idea of how the various incomes are distributed among the people. Such a general view cannot be readily gained from the entire unorganised collection of data.

<table>
<tr><th>Class</th><th>Tally</th><th>No. of games</th></tr>
<tr><td>0 – 9</td><td>||||</td><td>4</td></tr>
<tr><td>10 – 19</td><td>卌 |</td><td>6</td></tr>
<tr><td>20 – 29</td><td>卌 ||</td><td>7</td></tr>
<tr><td>30 – 39</td><td>卌 ||</td><td>7</td></tr>
<tr><td>40 – 49</td><td>卌 ||</td><td>7</td></tr>
<tr><td>50 – 59</td><td>|||| |</td><td>6</td></tr>
<tr><td>60 – 69</td><td>|||</td><td>3</td></tr>
<tr><td>70 – 79</td><td>|||</td><td>3</td></tr>
<tr><td>80 – 89</td><td>|||</td><td>3</td></tr>
<tr><td>90 – 99</td><td>|</td><td>1</td></tr>
<tr><td>100 – 109</td><td>|||</td><td>3</td></tr>
<tr><td>Total</td><td></td><td>50</td></tr>
</table>

Now we can easily answer the questions.

Let's look at another situation

The weights of the members of the school Health club are given below; in kilograms.

38	$37\frac{1}{2}$	$40\frac{1}{2}$	59	48	48	$37\frac{1}{2}$
58	50	$54\frac{1}{2}$	39	40	$40\frac{1}{2}$	49
32	43	45	53	37	44	51
$50\frac{1}{2}$	$32\frac{1}{2}$	46	55	36	$44\frac{1}{2}$	47
$42\frac{1}{2}$	33					

We want to make a frequency table.

Would classes like 30 – 34, 35 – 39, 40 – 44, 45 – 49 nd so on do?

In which class would we put $44\frac{1}{2}$ kilograms, for example.

We can take classes as 30 – 35, 35 – 40, 40 – 45 and so on $44\frac{1}{2}$ can be put in the class 40 – 45.

Classification methods

We classify and tabulate data for a concise presentation, from which it is easy to draw general conclusions. We have noted that some information is lost when we do this. This loss can be reduced by forming a large number of classes with small widths. But then the table will not be concise. On the other hand, if we form a few classes of large widths, the presentation would be compact, but the loss of information would be so great that no valid inferences could be drawn.

For example, in our example on incomes, suppose we divide the incomes into classes of width 1 rupee. All the collected information would be in the table; but there is no condensation of data. At the other extreme, if we consider the entire range of incomes as a single class, from the lowest income to the highest, then we have maximum condensation; but no general conclusions can be drawn from it.

But then, in which class would we put 40? 35 – 40 or 40 – 45. Usually, it is put in the class 40 – 45. Likewise, 45 is put the 45 – 50 class.

Now we can make a frequency table:

Class	Tally	Frequency
30 – 35		
35 – 40		
40 – 45		
45 – 50		
50 – 55		
55 – 60		

1) Given below are the highest temperatures (in degree Celsius) one day in 40 towns. Make a frequency table.

41	23	32	40	25	30	38	47	40	39
26	31	37	32	36	41	30	25	27	30
29	40	38	36	43	37	28	27	32	36
38	36	33	32	28	27	23	26	28	31

2) The heights (in centimeters) of 45 people who look part in a physical fitness test are given below.

Make a frequency table.

160	145	168	156	168.4	170	163	177	143	175	169	154
163	176	160.3	164	150	168	166	148	154	159	164.5	
165	155	148.2	158	174	169	168	165	170	141	172.7	
179	167	171	159	167	171	165	171	167	162	171	

Height	Tally	Number
140 – 145		
145 – 150		

A new picture

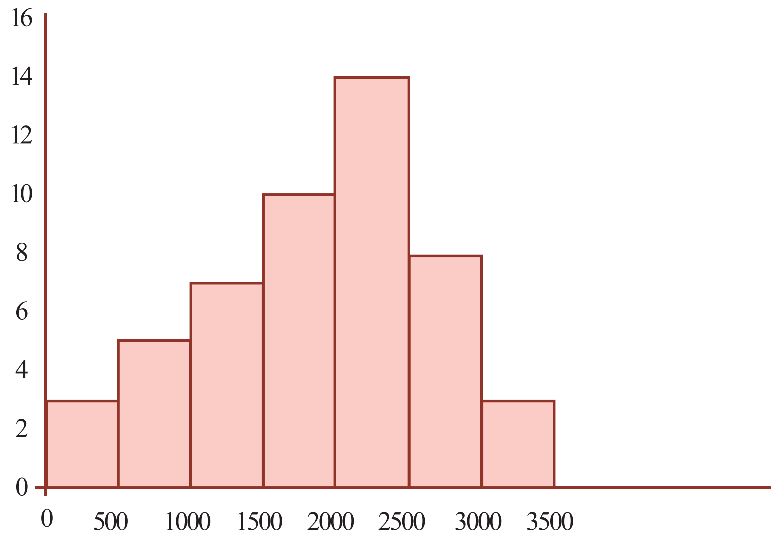
We have seen how numerical data can be pictorially represented as bar charts or pie diagrams.

Now let' see how the data given in a frequency table can be represented by a picture.

The table below gives the amount of water 50 households use.

Amount of water (litres)	Number of households
0 – 500	3
500 – 1000	5
1000 – 1500	7
1500 – 2000	10
2000 – 2500	14
2500 – 3000	8
3000 – 3500	3
Total	50

See how this data is represented by a picture:



The classes are marked on the horizontal line and frequencies on the vertical line. The width of each rectangle shows the length of the class interval and its height shows the frequency. Such a picture is called a histogram.

1) The table shows the times 30 children took to complete a long distance race. Draw a histogram of this.

Time (min)	Number of children
10 – 13	2
13 – 16	5
16 – 19	12
19 – 22	8
22 – 25	3

2) The table shows the daily incomes of 60 households in a locality.

Daily income (Rs.)	Number of households
200 – 250	3
250 – 300	7
300 – 350	15
350 – 400	20
400 – 450	9
450 – 500	6

Draw a histogram.

3) Detail of rainfall in June and July are given in the table below. Draw a histogram.

Rainfall (mm)	Days
10 – 20	4
20 – 30	6
30 – 40	9
40 – 50	15
50 – 60	10
60 – 70	8
70 – 80	5
80 – 90	3
90 – 100	1

4) The time taken by 25 women and 23 men to complete a race are given in the table below. Draw separate histograms for men and women.

Time (sec)	Number	
	Women	Men
30 – 40	2	3
40 – 50	6	7
50 – 60	8	5
60 – 70	5	5
70 – 80	4	3

5) The weights of 45 children in a class are listed below.

41, 31, 48, 34, 75, 39, 45, 41, 55

52, 40, 57, 43, 61, 47, 64, 56, 47

41, 59, 46, 67, 45, 64, 48, 52, 58

53, 64, 59, 43, 50, 62, 54, 68, 59

69, 57, 57, 53, 52, 56, 61, 55, 69

Make a frequency table and draw a histogram.

Looking back

Learning outcomes	What I can	With teacher's help	Must Improve
• Making a frequency distribution of individual entries from given data			
• Dividing given data into classes and make a frequency table.			
• Explaning the need for grouping into classes in making a frequency table.			
• Representing a frequency table as a histograms.			