

MATHEMATICS

Part - 2

Standard
VI



Government of Kerala
Department of General Education

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THE NATIONAL ANTHEM

Jana-gana-mana adhinayaka, jaya he
Bharatha-bhagya-vidhata
Punjab-Sindh-Gujarat-Maratha
Dravida-Utkala-Banga
Vindhya-Himachala-Yamuna-Ganga
Uchchala-Jaladhi-taranga
Tava subha name jage,
Tava subha asisa mage,
Gahe tava jaya gatha
Jana-gana-mangala-dayaka jaya he
Bharatha-bhagya-vidhata
Jaya he, jaya he, jaya he,
Jaya jaya jaya, jaya he.

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give my parents, teachers and all elders, respect and treat everyone with courtesy.

To my country and my people, I pledge my devotion. In their well-being and prosperity alone, lies my happiness.

Mathematics

6

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Dear children,

We have already learned about different dimensions of Mathematics. We also have gained a clear understanding of counting numbers, various operations, basic concepts of geometric shapes, number relations, fractions and decimal forms in previous classes. The content of this book will help us to apply the concepts we have learned and create new ideas. These lessons will guide us to increase our Mathematical skills, solve problems logically and understand Mathematics based on observations and conclusions. Let's confidently enter into the fascinating and curious world of Mathematics.

With best wishes,

Dr. Jayaprakash R.K.

Director,
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The icon below is used in this
textbook for convenience



Let's do the problems

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a ¹**[SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC]** and to secure to all its citizens :

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the ²[unity and integrity of the Nation];

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949 do **HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**

-
1. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec. 2, for "Sovereign Democratic Republic" (w.e.f. 3.1.1977)
 2. Subs. by the Constitution (Forty-second Amendment) Act, 1976, Sec. 2, for "Unity of the Nation" (w.e.f. 3.1.1977)

MONEY MATH



Discount

See this ad:



30 percent discount means what?

It means when we buy clothes worth 100 rupees, we can deduct 30 rupees and need pay only 70 rupees

So, what if we buy clothes for 200 rupees?

200 is two times 100, right? So, the deduction also is doubled.

That means $2 \times 30 = 60$ rupees

So we need pay only $200 - 60 = 140$ rupees

Thus 30 percent deduction means subtracting 30 rupees for each purchase of 100 rupees

Word and meaning

The word *percent* comes from the Latin words *per* meaning “through” and *centum* meaning “hundred”. The word *per* is now used in English to mean “for each”. Thus *percent* means, “for each hundred”.



Now can you fill in the table below by computing the deduction for each purchase and the amount to be paid?

Price (Rupees)	Deduction (Rupees)	Amount to be paid (Rupees)
500	$5 \times 30 = 150$	$500 - 150 = 350$
1000		
1200		

Interest

See this news item:



If we deposit money in a bank or other financial institutions, we get a bit more than the amount we deposited, when we withdraw it after some time. This extra amount is called *interest*.

In the picture above, the symbol 9% is a shorthand for 9 percent.

So it means, for every 100 rupees we deposit, we get 9 rupees as interest after a year; thus we get $100 + 9 = 109$ rupees in all.

If we deposit 1500 rupees now, how much would we get back after one year, according to this?

We can calculate the interest, just as we calculated the discount earlier.

- (i) The interest for one year is 9 rupees for each 100 rupees.
- (ii) 1500 is 15 times 100.
- (iii) The interest for 1500 rupees is $15 \times 9 = 135$ rupees.

Thus after one year, we would get this added to 1500 rupees, which is $1500 + 135 = 1635$ rupees.

See the amounts some people have deposited in this bank:

Ash	15000 Rupees
Suma	25000 Rupees
Niharika	37500 Rupees
Vihan	12500 Rupees
Ahi	22500 Rupees



Calculate the interest and the total amount each would get back after one year.

On the other hand, if we take a loan from a bank, we would have to pay interest to the bank. This also is usually said as a percent, so much for every hundred rupees each year.

See this problem:

A bank gives agricultural loans. On repayment after a year, 6% interest also has to be paid. How much should a person who borrows 25000 rupees now, have to repay after a year?

Let's calculate the interest first, as we did earlier:

- (i) The interest each year for 100 rupees is 6 rupees
- (ii) 25000 rupees is 250×100
- (iii) Interest is $250 \times 6 = 1500$

So, the amount to be repaid is $25000 + 1500 = 26500$ rupees



- (1) See the board in front of the shop:



- Sreesha bought cloths worth 3500 rupees from this shop. How much should she pay?
- (2) Angelo Tomy borrowed 45000 rupees from a bank. The interest is 12%. How much should he pay back after one year?
- (3) Mehul contributes 8% of his earnings each month to medical aid for the poor. His income this month is 75000 rupees. How much is his contribution this month?
- (4) Eugene saves 15% of his income each month. His income in January was 64000 rupees. How much did he save that month?
- (5) A TV manufacturer decides to reduce the price by 5% from the next month. What will be the price next month of a TV which costs 45000 rupees now?
- (6) A car manufacturer decides to increase the price by 2% from the next month. What will be the price next month of a car which costs 560000 rupees now?

Percent problems

Let's look at another discount problem like the one we did earlier.

A textile shop offers 30% discount. If clothes are bought for 450 rupees, what is the actual discount amount?

We can easily calculate the discount on 400 rupees, can't we?

Since $400 = 4 \times 100$, the discount is $4 \times 30 = 120$ rupees.

Now, how much is the discount on the remaining 50 rupees?

50 is half of 100, right?

So, the discount is half of 30, which is 15.

Thus, we can calculate the total discount on 450 rupees as the sum of the discount 120 for 400 rupees and the discount 15 for 50 rupees, which makes $120 + 15 = 135$ rupees.

What if 475 rupees worth of clothes are bought?

We have already calculated the discount on 450 rupees. So it only remains to calculate the discount on 25 rupees.

We think like this:

- (i) 25 is half of 50
- (ii) The discount on 50 rupees is 15 rupees
- (iii) The discount on 25 rupees is half of 15 rupees, which is 7 rupees and 50 paise

The total discount can be calculated like this:

discount on 400 rupees	120 rupees
discount on 50 rupees	15 rupees
discount on 25 rupees	7 rupees 50 paise
discount on 475 rupees	142 rupees 50 paise

Can we calculate the discount on 460 rupees like this?



We have calculated the discount on 400 rupees. What remains is the discount on 60 rupees. How do we calculate the discount on this?

- (i) discount on 100 rupees is 30 rupees
- (ii) discount on 10 rupees is $30 \div 10 = 3$ rupees
- (iii) discount on 60 rupees is $6 \times 3 = 18$ rupees

Thus the discount on 460 rupees is $120 + 18 = 138$ rupees.

What if clothes worth 468 rupees are bought?

For that we must calculate the discount on 8 rupees.

We just now calculated the discount on 10 rupees as 3 rupees. Let's write it as paise and continue like this:

- (i) Discount on 10 rupees is 300 paise
- (ii) Discount on 1 rupee is 30 paise
- (iii) Discount on 8 rupees is $8 \times 30 = 240$ paise
- (iv) That is 2 rupees 40 paise

So what is the total discount on 468 rupees?

How much should be actually paid?

Can't we calculate the discount on any amount like this?

For example what if we buy clothes worth 3546 rupees?

Let's write the calculations in a slightly shortened form

First we split 3546 into hundreds, tens and ones

$$3546 = 3500 + 40 + 6$$

The discount on 3500 rupees can be easily calculated:

$$35 \times 30 = 1050 \text{ rupees}$$

To compute the discount on 40 rupees, we use the fact that the discount on 10 rupees is 3 rupees, as seen in the last problem.

$$4 \times 3 = 12 \text{ rupees}$$

The discount on the remaining 6 rupees can be calculated as paise. We have seen in the last problem that the discount on 1 rupee is 30 paise.

$$6 \times 30 = 180 \text{ paise} = 1 \text{ rupee } 80 \text{ paise}$$

Adding all these, we find the total discount as 1063 rupees and 80 paise.

Now let's look at a problem on interest:

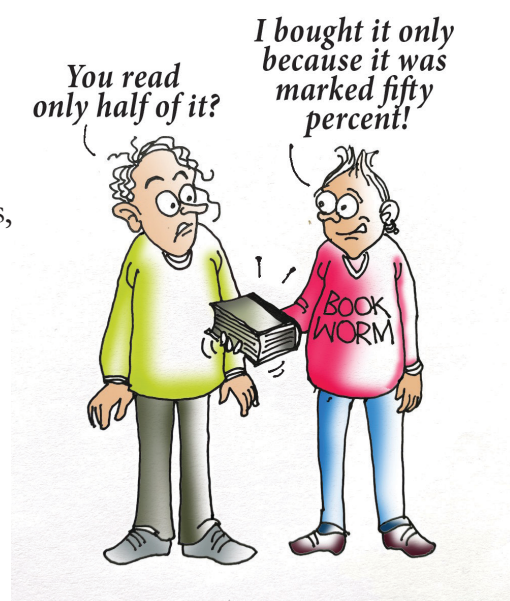
A person deposited 16725 rupees in a bank, which pays 9% interest.
How much will he get back after one year?

As before, first we split 16725 like this:

$$16725 = 16700 + 20 + 5$$

Then we compute the interest for each hundred rupees, ten rupees and one rupee like this:

Amount Rupees	Interest	
	Rupees	Paise
100	9	0
10	0	90
1	0	9



Now we can compute the interest for 16725 rupees like this:

	Amount Rupees	Interest	
		Rupees	Paise
$167 \times 9 = 1503$	16700	1503	0
$2 \times 90 = 180$	20	1	80
$5 \times 9 = 45$	5	0	45
Total	16725	1505	25



- (1) At the Books Festival, all books are given 15% discount. See the prices of books Sreena and her friends bought:

Sreena	540 Rupees
Tessy	375 Rupees
Nassar	630 Rupees
Jacob	432 Rupees

Calculate how much each has to pay.

- (2) A person borrowed 26750 rupees from a bank, where the interest is 12% each year. How much should he pay back after one year?
- (3) A company gives 8% of one month's salary as bonus for Onam. How much does a man, whose monthly salary is 46375 rupees, get as bonus?

What percent?

See this problem:

A shop offers a discount on all products, as a fixed percent. A person buying things worth 2000 rupees got a discount of 300 rupees. What is the discount percent?

What is given as discount percent is actually the amount deducted for every hundred rupees.

Here 300 rupees is deducted for 20 hundreds. So, how much is deducted for every hundred? We need only divide 300 by 20, right?

We have seen in Class 5 that in division, we can remove common factors (The section **Common factors** in the chapter **Within Numbers**) We have

$$300 = 30 \times 10 \text{ and } 20 = 2 \times 10$$

So that

$$300 \div 20 = 30 \div 2 = 15$$

Thus the deduction for every hundred rupees is 15 rupees; this means the discount is 15%.

Let's look at another problem:

When a saree worth 750 rupees was bought, 150 rupees was deducted from the price. What is the discount for clothes in this shop?

We cannot split 750 rupees in this problem exactly into hundreds. So, we think like this:

750 is 75 tens; and the discount on this is 150 rupees.

So, what is the amount deducted for every 10 rupees?

$$150 \div 75 = 30 \div 15 = 10 \div 5 = 2$$

Thus 2 rupees is deducted for every 10 rupees

So, 20 rupees is deducted for every 100 rupees. In other words, the discount is 20%.

One more problem,

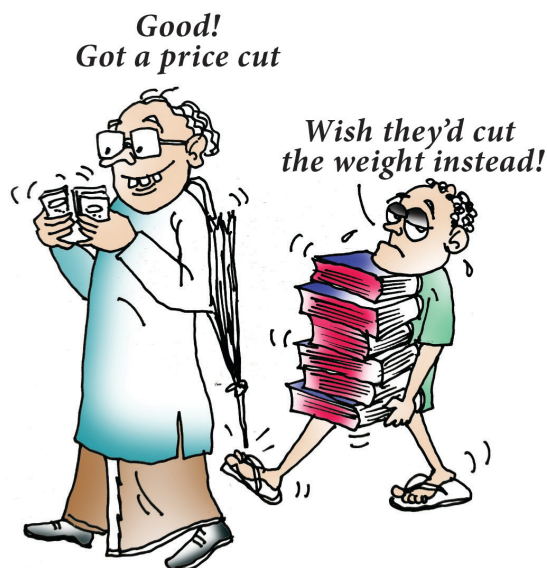
When books were bought for 450 rupees, a discount of 54 rupees was given. What percent is the discount?

It is not easy to calculate the discount for 100 rupees or 10 rupees, as in the earlier problem (Why?). Instead let's calculate the discount on 1 rupee as paise.

54 rupees is 5400 paise and this is the discount for 450 rupees.

So how do we compute the discount on 1 rupee as paise?

$$5400 \div 450 = 540 \div 45$$



Next let's factorize 540 and 45 like this

$$540 = 60 \times 9 \text{ and } 45 = 5 \times 9$$

So,

$$540 \div 45 = 60 \div 5 = 12$$

Thus the discount on each rupee is 12 paise.

This means 12 rupees for every 100 rupees.

Thus the discount is 12%.



- (1) 25000 rupees deposited in a bank earned 2000 rupees as interest in a year. What is the interest percent?
- (2) For books worth 480 rupees, a shop gave 72 rupees as discount. What is the discount percent?
- (3) A mobile phone manufacturer sells all the previous year's models at a fixed discount. A phone which cost 45000 rupees earlier is now sold for 37800 rupees. What is the discount percent?

UNCHANGING RELATIONS

Unchanging sum

Mayukhi has 100 rupees in her money box. She took 10 rupees from it and bought a pen. How much is left?

What if she had spent 20 rupees instead?

If she has 75 rupees left, how much did she spend?

What is the relation between the money spent and the money left? This can be said in different ways, right?



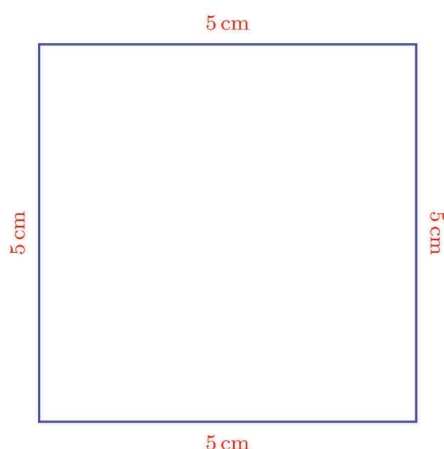
- The money spent subtracted from 100 rupees gives the money left.
- The money left subtracted from 100 rupees gives the money spent.
- The sum of the money spent and the money left is 100 rupees.

What we said last can be said in little more detail:

The money spent and the money left can change, but their sum does not change.

Let's look at another problem: we want to draw a square of perimeter 20 centimetres.

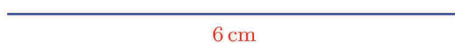
To draw a square, we need the length of a side. How long is it?



Now can you draw a rectangle (not a square) of the same perimeter?

What should be the lengths of the sides?

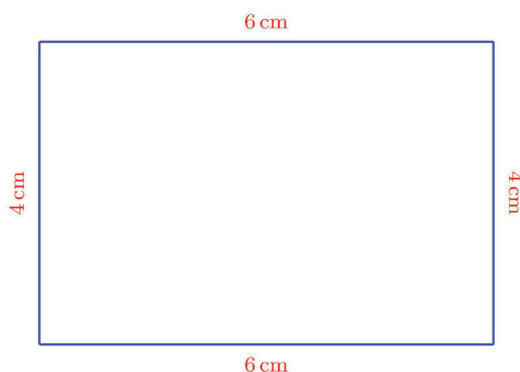
Let's start by extending the bottom side of the square by 1 centimetre:



How tall should be the vertical side?

We have seen in Class 5 that the perimeter of a rectangle is twice the sum of its width and height (the section **Calculations** in the lesson **Rectangle Math**).

Here the perimeter is 20 centimeters, so that the sum of the width and height is 10 centimetres; since the width is 6 centimetres, the height should be $10 - 6 = 4$ centimetres:



We can draw many other rectangles with the same perimeter, can't we?

What is the relation between the width and height in all these?

The width and height of the rectangle can change; but not their sum.

In the money problem, the amount spent and the amount left can change, but not their sum. In the rectangle problem, height and width can change, but not their sum.

We can note another thing.

In the money problem, if the money spent is increased by 1 rupee, the money left is decreased by 1 rupee.; if the money spent is decreased by 5 rupees, then the money left is increased by 5 rupees.

In short,

However more is the money spent, that much less is the money left; however less is the money spent, that much more is the money left.

What about the rectangle problem?

However longer is the width, that much shorter is the height; however shorter is the width, that much longer is the height.

Now see this problem:

128 rupees was spent and 99 rupees is left. How much money was there at first?

We need just add 128 and 99, right?

Instead, we can also think like this:

- (i) If the money spent is 1 rupee less, then the money left is 1 rupee more
- (ii) If the money spent is 127 rupees, the money left would be 100 rupees
- (iii) The money originally had is $127 + 100 = 227$ rupees

This can be written as an operation of numbers alone:

$$128 + 99 = 127 + 100 = 227$$

We can write this operation as a general principle:

The sum of two numbers is the same as the sum of the numbers got by adding a number to one of them and subtracting the same number from the other.



(1) Mentally calculate the sums below:

(i) $137 + 199$ (ii) $375 + 298$ (iii) $697 + 174$ (iv) $1489 + 2363$

(2) (i) Take any four consecutive natural numbers. Calculate the sum of the first and the last, and also the sum of the middle two numbers. Take other four consecutive natural numbers and check. Why are the sums the same every time?

(ii) Instead of four consecutive natural numbers, take six consecutive natural numbers. The sum of the first and the last numbers is equal to which other pairs? Why is this so?

(iii) What if we take five consecutive natural numbers?

(iv) What about three consecutive natural numbers?

(3) In the calendar of any one month, mark those dates which fall on any one day of the week:

SUN	MON	TUE	WED	THU	FRI	SAT
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

The sum of the first and last numbers is equal to the sum of which other pairs? Why?

Unchanging difference

Mary is now 24 years old and her brother Johnny is 28 years old.
After 2 years, what will be Mary's age? And
Johnny's age?

What were their ages 3 years back?

What can we say in general about their
ages?

The ages of both change; but the
difference in ages does not change.

In other words, after some years, Johnny's age
will increase; and Mary's age will increase by the
same number. What about some years back?

What we saw here can also be stated as a general
principle:

*He's twelve now?
How time flies!*

*Hmm...
How many years
did it take for you
to be twelve?*



The difference of two numbers is the same as the difference of the two numbers got by adding or subtracting the same number to each of them

For example, see this problem:

$$285 - 198$$

Let's add 2 to each number:

$$285 + 2 = 287$$

$$198 + 2 = 200$$

So, by the principle above,

$$285 - 198 = 287 - 200 = 87$$

Here's another one:

$$600 - 358$$

Let's subtract 1 from each now:

$$600 - 1 = 599$$

$$358 - 1 = 357$$

And then we have

$$600 - 358 = 599 - 357 = 242$$



Calculate these differences in your head:

- (i) $453 - 196$ (ii) $388 - 189$ (iii) $700 - 387$ (iv) $1500 - 839$

Let's look at another problem.

The difference of any two even numbers is again an even number. isn't it?

For example,

$$18 - 12 = 6$$

Is it true for all even numbers? Why?

In the above example, we can write 18 and 12 as multiples of 2:

$$18 = 9 \times 2$$

$$12 = 6 \times 2$$

Then their difference can be written like this:

$$18 - 12 = (9 \times 2) - (6 \times 2)$$

We have seen in Class 5 that multiplying two numbers by the same number and then subtracting the products gives the same answer as multiplying the difference of the numbers by the same number (the section **Addition, subtraction and multiplication** in the lesson, **Number Relations**)

Using this, we can write the difference above like this:

$$\begin{aligned} 18 - 12 &= (9 \times 2) - (6 \times 2) \\ &= (9 - 6) \times 2 \\ &= 3 \times 2 \end{aligned}$$

Can't you see that in this computation, even if we take other even numbers instead of 18 and 12, we still get the difference as a multiple of 2?

Thus we have

The difference of two even numbers is an even number.

Now take a pair of odd numbers and find the difference. Isn't the difference an even number?

Why is it so?

Every odd number is 1 added to an even number, right?

For example

$$325 = 324 + 1$$

$$137 = 136 + 1$$

What is their difference?

According to the general result above, the difference of two numbers is the same as the difference between the numbers got by subtracting 1 from each. That is,

$$325 - 137 = 324 - 136$$

We have already seen that the difference of two even numbers is even. So, without actually subtracting, we can see that $324 - 136$ is an even number; and this is the same as $325 - 137$

Like this, we can rewrite the difference of any two odd numbers as the difference of two even numbers and see that the difference is an even number



- (1) Is the difference of any two multiples of 3 again a multiple of 3? Why?
- (2) (i) If two numbers leave remainder 1 on division by 3, is their difference a multiple of 3? Why?
- (ii) If two numbers leave remainder 2 on division by 3, is their difference a multiple of 3? Why?

- (3) What all things can we say if we take 4 instead of 3 in the last problem?
- (4) If two numbers leave the same remainder on division by a number, what is the relation between the difference of these numbers and the dividing number?

Unchanging product

Rafi has 120 rupees and he wants to buy some notebooks.

How many 15-rupee notebooks can he buy?

What about 30-rupee notebooks?

When the price of a notebook is doubled, what happens to the number of notebooks?

Now if he wants 6 notebooks, what is the price of a notebook he can buy?

What if he wants just two notebooks?

What do we see here?

- The price of a notebook and the number of notebooks can change; but their product should be 120.
 - * 8 notebooks priced 15 rupees can be bought.
 - * 4 notebooks priced 30 rupees can be bought.
- Doubling the price of a book halves the number of notebooks.
 - * If he wants to buy 6 notebooks, he can buy 20-rupee notebooks.
 - * If he wants only 2 notebooks, he can buy 60-rupee notebooks.
- If the number of books is made one-third, the price of a notebook can be tripled.

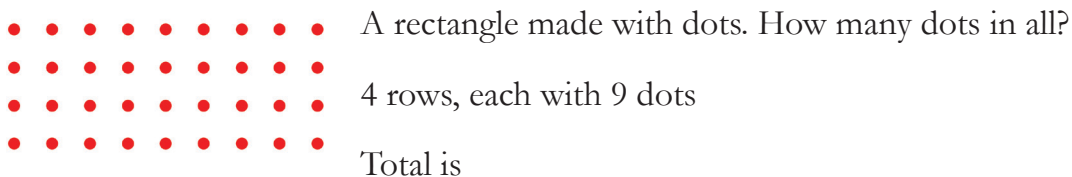
What can we say in general?

If one of the two numbers, the price of a notebook and the number of notebooks, is multiplied by a number, the other should be divided by the same number.



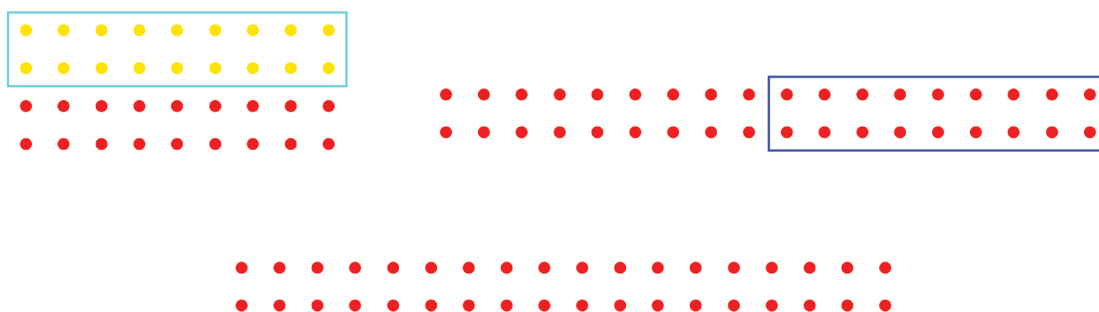
Let's look at another example

See this picture:



$$4 \times 9 = 36$$

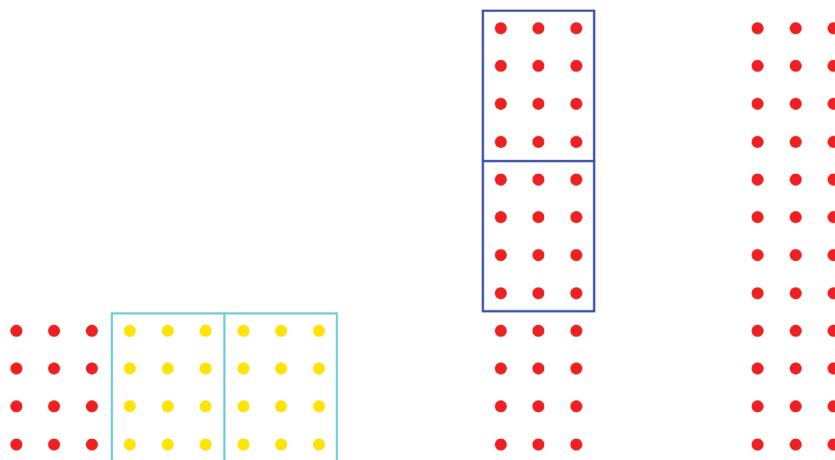
Now suppose we remove the top two rows and put them on the right:



The number of rows is halved; and the number of dots in each row is doubled.
How many dots in all?

$$2 \times 18 = 36$$

What if we cut the original rectangle into three pieces vertically and stack two of them on top of the third?



The number of rows is $3 \times 4 = 12$; and each row has $9 \div 3 = 3$ dots

What about the number of dots?

$$12 \times 3 = 36$$

Here, the number of rows and the number of dots in each row change, but their product is unchanged; that is, the total number of dots don't change.

When the number of rows is halved, the number of dots in each row is doubled. If the number of dots in each row is made one-third, the number of rows is tripled.

So, as in the case of the notebook problem, here also, if one of two numbers is multiplied by a number and the other is divided by the same number, the product does not change.

What we saw in these two examples can be stated as a general principle:

The product of two numbers is the same as the product of the numbers got by multiplying one of them by a number and dividing the other by the same number

For example, let's compute

$$18 \times 5$$

Let's divide 18 by 2 and multiply 5 by 2:

$$18 \div 2 = 9$$

$$5 \times 2 = 10$$

Then we have

$$18 \times 5 = 9 \times 10 = 90$$

Remember doing such problems in Class 5? (The section, **Factors in multiplication and division** of the lesson **Within Numbers**).

In the section **Unchanging sums** we saw instances of the sum of two numbers being unchanged, even though the numbers themselves changed. There, when one number was made larger, the other had to be made smaller. In what way? Whatever number added to one of these must be subtracted from the other.

What we see here are instances of the product of two numbers being unchanged, even when the numbers change. Here also, when one number was made larger, the other had to be made smaller. In what way? Whatever number by which we multiply one number, the other must be divided by the same number.



- (1) If we start with 1 and double again and again, we get the numbers 1, 2, 4, 8, 16, ...
 - (i) Take any four consecutive numbers from these. Why is the product of the first and last numbers the same as the product of the middle two numbers?
 - (ii) If we take three consecutive numbers from these, what is the relation between the product of the first and last numbers and the middle number?
- (2) If we start with 1 and triple it again and again, we get the numbers 1, 3, 9, 27, 81, ... For these also, are the results of the first problem true? Why?

Unchanging quotients

There are 40 children in a class and all of them are to be given 5-rupee pens. How much money is needed?

What if such pens are to be bought for 80 children?



How does the money to be spent changes when the number of pens is doubled?

With 50 rupees how many pens can be bought?

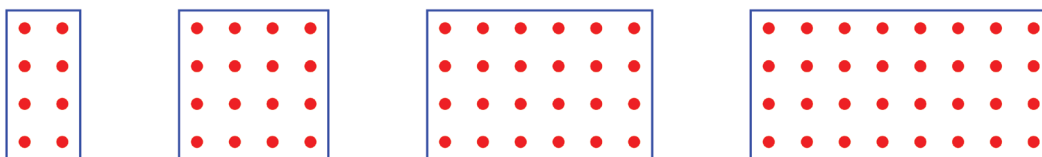
And with 150 rupees?

When the money to be spent is tripled, what is the change in the number of pens?

Here, the number of pens and the money spent changes. What remains unchanged?

Thus in this instance, the number of pens and the money spent change; but on dividing the money to be spent by the number of pens, we get 5 always; and that is the price of one pen.

Now look at these pictures:



Four rectangles made up of dots.

The number of dots in each row and the total number of dots change.

What remains unchanged?

The number of dots in each row of the second rectangle is how many times the number of dots in each row of the first?

And what is the total number of dots in the second rectangle?

How do these numbers change from the first to the third rectangle?

In the pen problem and the rectangle problem, two numbers changed into the same multiples; but the quotients do not change.

From this, what can we say about the quotient of two numbers?

The quotient of two numbers is the same as the quotient of these two numbers multiplied by the same number.

We have seen in Class 5 that the quotient of two numbers do not change if both of them are divided by the same number, instead of being multiplied by the same number. (The section **Common factors** of the lesson **Within Numbers**).

For example, recall how we divided 210 by 42:

(i) $210 \div 42$

• $210 \div 2 = 105$ $42 \div 2 = 21$

(ii) $210 \div 42 = 105 \div 21$

• $105 \div 3 = 35$ $21 \div 3 = 7$

(iii) $105 \div 21 = 35 \div 7$

(iv) $35 \div 7 = 5$

(v) $210 \div 42 = 5$

This was stated as a general principle like this in Class 5:

In dividing one number by another, common factors can be removed.

We can also state it like this:

The quotient of two numbers is the same as the quotient of these two numbers divided by the same number.

This together with what we said earlier can be stated like this:

The quotient of two numbers is the same as the quotient of these two numbers multiplied or divided by the same number.

We have seen many examples of dividing two numbers by the same number to find their quotient easily, in Class 5. In some problems, multiplying both numbers by the same numbers simplifies the calculation of the quotient.

For example, to divide 135 by 5, we can multiply each by 2 and do like this:

$$\begin{aligned} 135 \div 5 &= 270 \div 10 \\ &= 27 \end{aligned}$$

Sometimes multiplication by the same number and division by the same number can be combined to make the computation easier. For example the division of 315 by 35 can be done like this:

$$\begin{aligned} 315 \div 35 &= 630 \div 70 \\ &= 63 \div 7 \\ &= 9 \end{aligned}$$

In the section **Unchanging differences** we saw instances of the difference of two numbers remaining unchanged, even though the numbers themselves change. There, when one number was made larger, the other also had to be made larger; and when one number was made smaller, the other also had to be made smaller. In what way? Whatever number added to one of these must be added to the other; and whatever number subtracted from one number must be subtracted from the other.

What we see here are instances of the quotient of two numbers being unchanged, even when the numbers change. Here also, when one number was made larger, the other had to be made larger; and when one number is made smaller the other must be made smaller. In what way? Whatever number by which one number is multiplied, the other must be multiplied by the same number; and whatever number by which one number is divided, the other number also must be divided by the same number.



Calculate the quotients below:

(i) $345 \div 15$ (ii) $495 \div 45$ (iii) $325 \div 25$ (iv) $975 \div 75$ (v) $875 \div 125$

LINES AND ANGLES



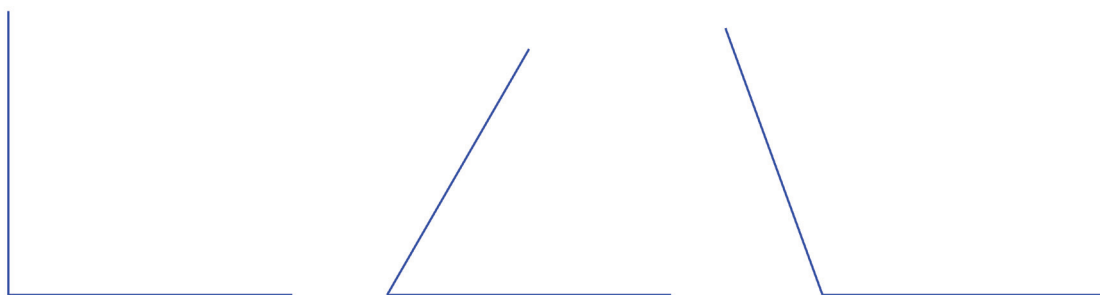
When lines meet

You have learned how to draw angles and measure them.

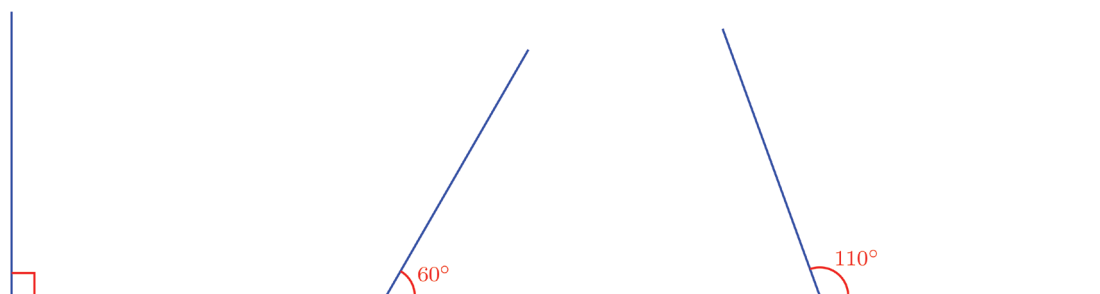
If we draw a line and draw another line from one of its ends, we get an angle:



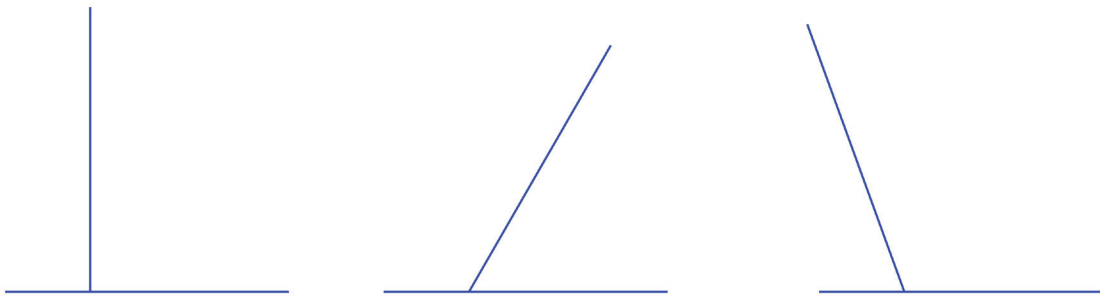
If the second line is drawn straight up, we get a right angle; if it is slanted towards one side, an angle smaller than a right angle, and if it is slanted towards the other side, an angle larger than a right angle.



And if we measure them? Right angle is 90° .

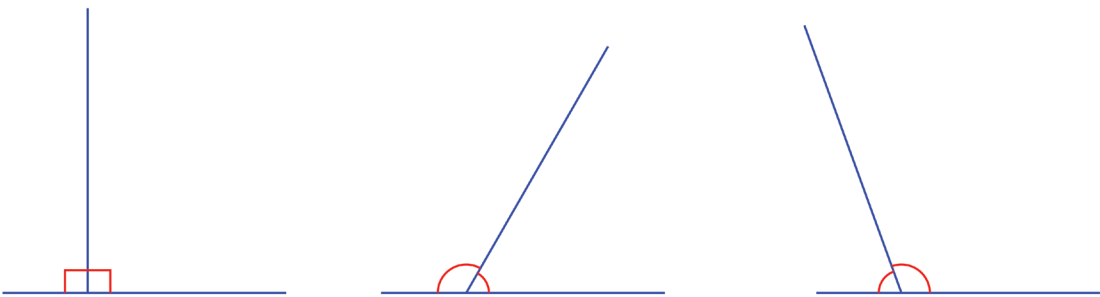


Now let's draw a line and then draw another line not from one of its ends, but from some other point on it; can be straight up, can be slanted



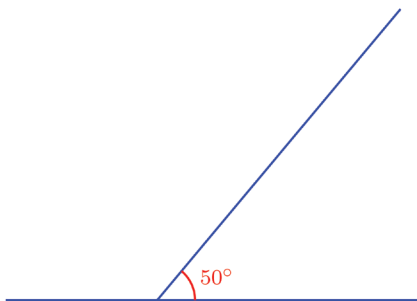
However we draw, we get two angles, on either side of the second line.

If the line is straight up, right angles on both sides. If it is slanted, less than a right angle on one side and more than a right angle on the other.



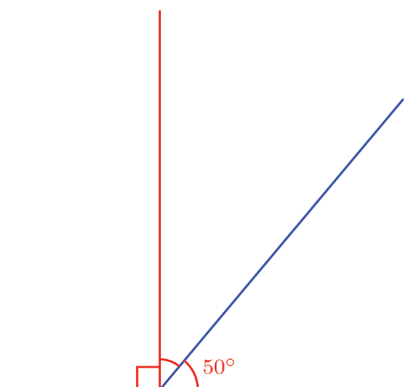
For a slanted line, is there any relation between the two angles on either side?

Draw a line and at some point on it, draw a 50° angle, as shown below:



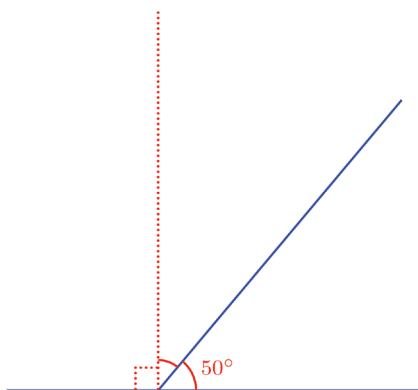
How much less than a right angle is this?

Let's draw a perpendicular as shown below:

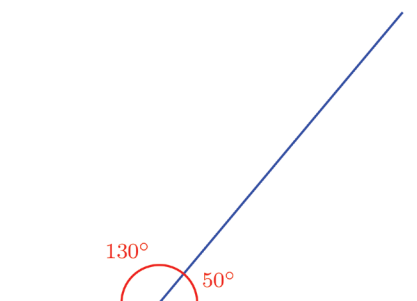


What is the angle between the slanted line and the perpendicular?

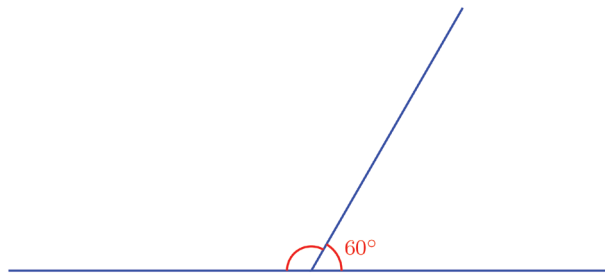
So, how much more than a right angle is the angle on the other side?



Now can you say how much the angle on the other side is?



Now see this picture:



How much is the angle on the left of the slanted line?

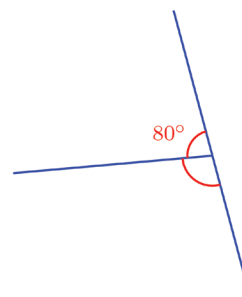
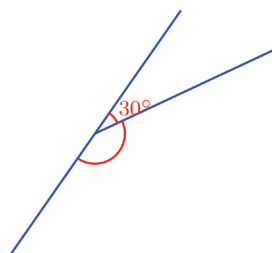
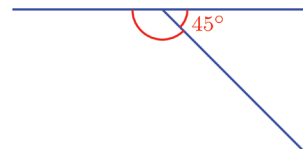
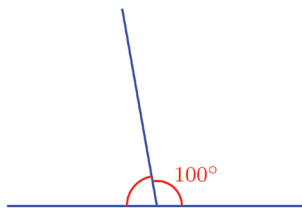
Imagine a perpendicular from the point where the lines meet.

How much less than a right angle is the angle on the right?

How much more than a right angle is the angle on the left?

So, how much is the angle on the left?

Some pictures of two lines which meet are shown below. The measure of one of the two angles made on either side is also given. Find the measure of the other angle:



What did we see in all these problems?

- (i) When two lines meet, two angles are made
- (ii) If the lines are perpendicular to each, then both the angles are right angles
- (iii) Otherwise, one angle is less than a right angle and the other is more than a right angle
- (iv) The larger angle is as much more than a right angle as the smaller angle is less than a right angle

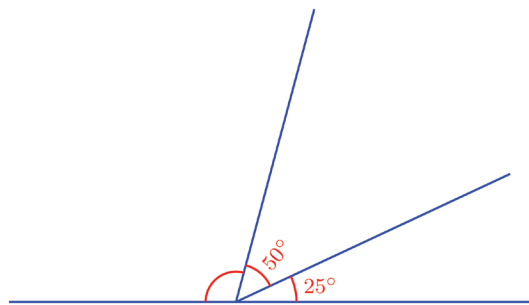
So, however the lines may be, how much is the sum of the two angles?

What can we say in general about the two angles made when two lines meet?

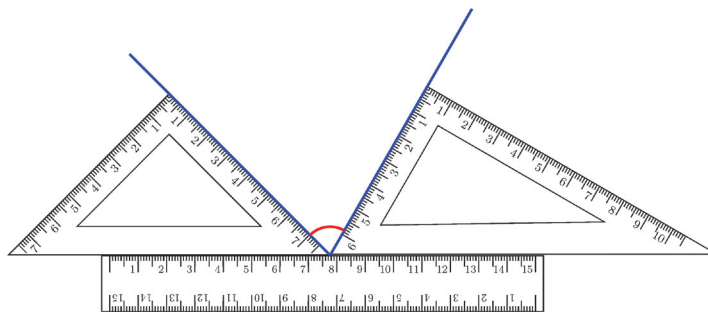
The sum of the two angles made when two lines meet, is 180°



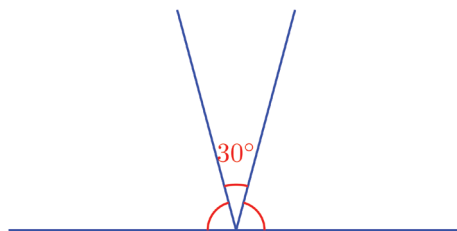
- (1) In the picture below, how much is the angle on the left?



- (2) In the picture below, what is the measure of the angle between the lines?



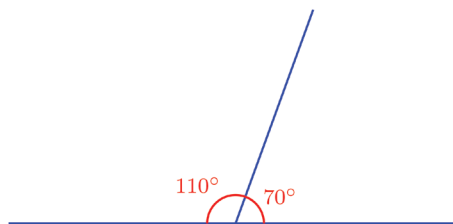
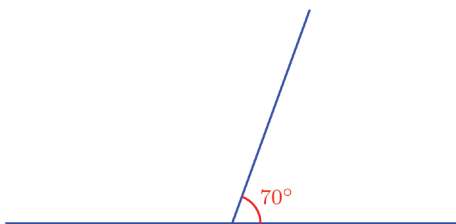
- (3) Among the three angles marked in the picture below, the angles on the left and right are equal. How much is each?



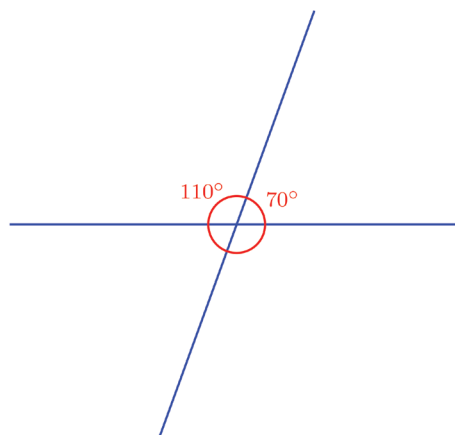
- (4) Of the two angles made when two lines meet, one angle is double the other. What are the measures of the two angles?

When lines intersect

We have seen that when a line meets another line, two angles are formed, and if we know one of these, the other can be calculated:

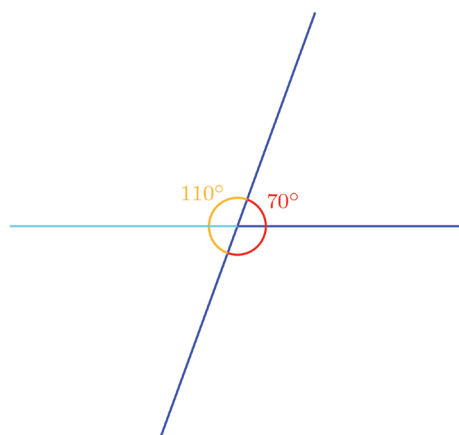


What if we extend the slanted line downward, cutting the horizontal line?



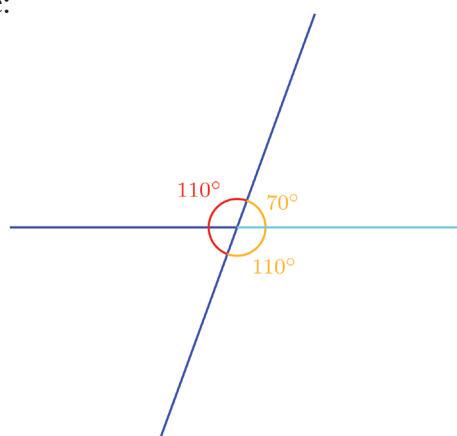
Now we have two angles at the bottom also. What are their measures?

Let's first calculate the bottom-right angle. For that, let's look at just the right part of the horizontal line and the slanted line:

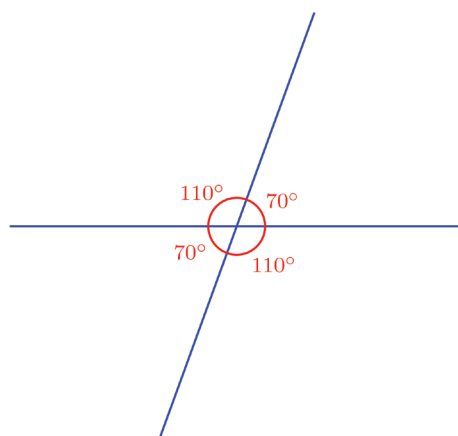


Of the two angles at the bottom, we now get the angle on the right, don't we?

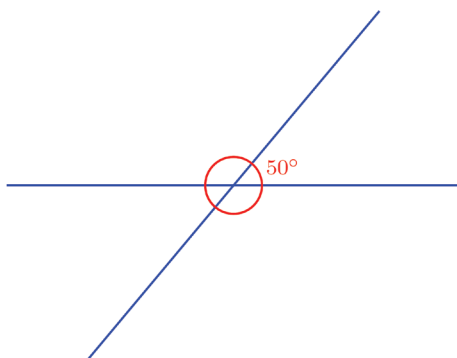
Like this, we can get the angle on the left by looking at just the left part of the horizontal line and the slanted line:



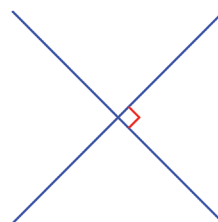
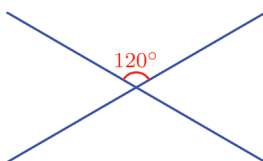
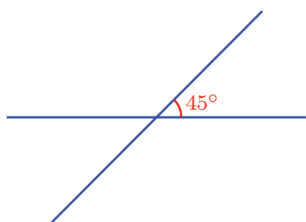
Thus we have all the four angles made by the two intersecting lines:



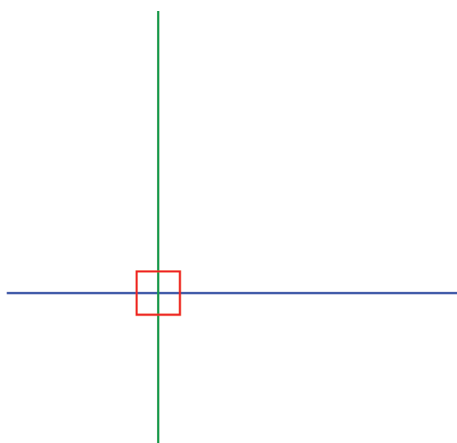
Can you now calculate the other three angles in this picture?



The pictures below show various pairs of intersecting lines. One of the four angles thus made is given in each picture. Calculate the other three angles and mark them in each picture:

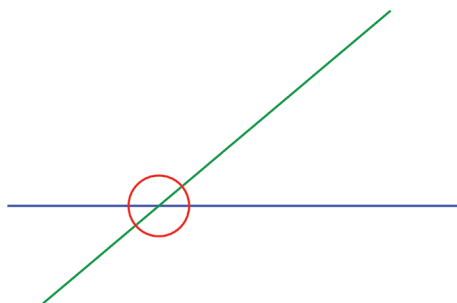


What can we say in general about the four angles made by two intersecting lines? If the lines are perpendicular to each other, there is not much to say; all four are 90° angles:



If one line is a bit slanted?

We get two small angles and two large angles. What is the relation between them?

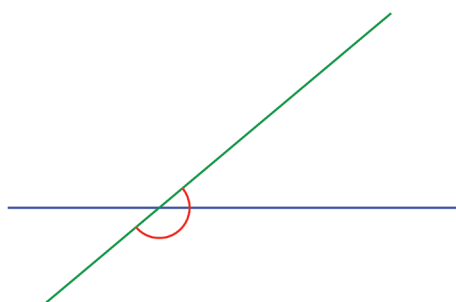
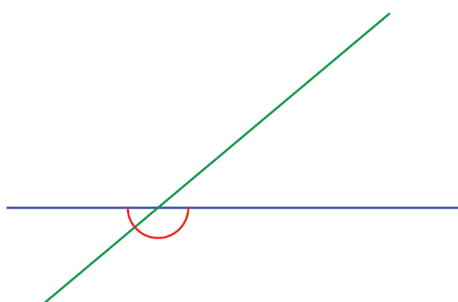
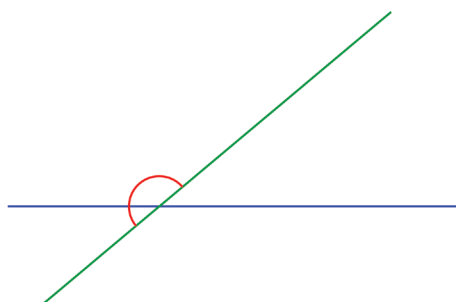
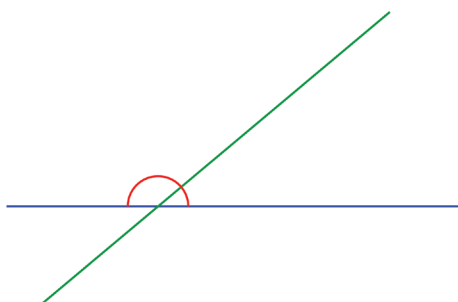


When two lines intersect, four angles are formed. If the lines are perpendicular to each other, all four are 90° angles. If the lines are slanted, there are two small angles and two large angles; in these,

- Both the smaller angles have the same measure
- Both the larger angles have the same measure
- The sum of a small angle and a large angle is 180°

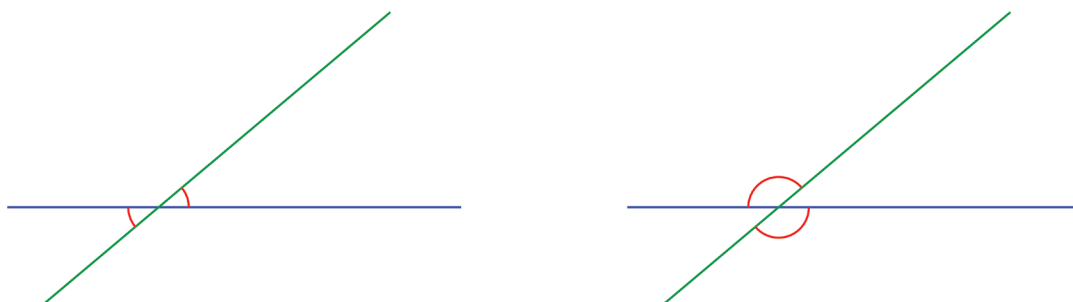
We can say it in a slightly different way

The four angles made by two intersecting lines can be paired in various ways. Look at these four pairs:



In each pair, the angles are next to each other. They are often called adjacent angles.

Now look at these two pairs:



In each pair, one angle is opposite the other. They may be called opposite angles.

Using these terms, the relations between the angles can be put like this:

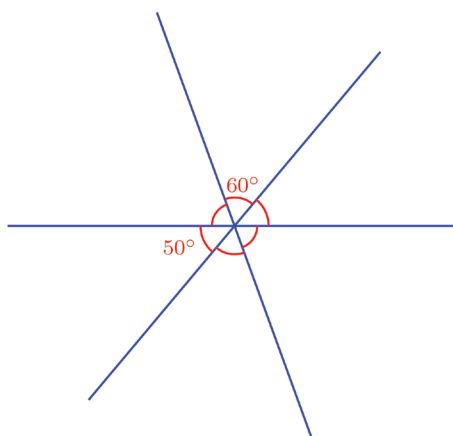
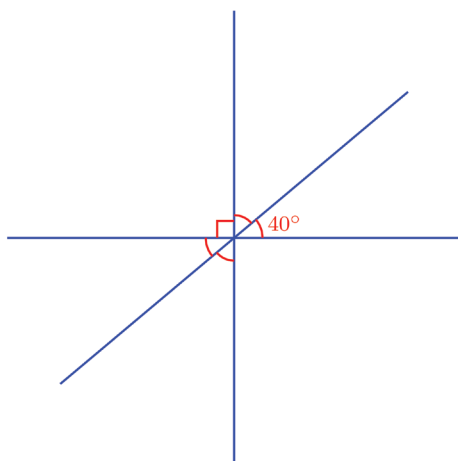
In the four angles made by two intersecting lines,

- The sum of each pair of adjacent angles is 180°
- Each pair of opposite angles are equal



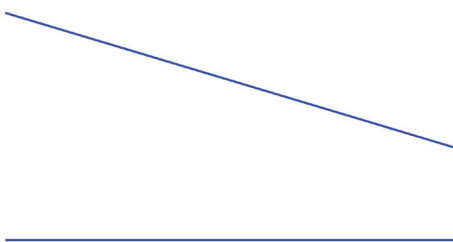
- (1) In the four angles made by two intersecting lines, the sum of two angles is 100° . Calculate all the four angles
- (2) In the four angles made by two intersecting lines, one angle is half of another. Calculate all the four angles

- (3) The two pictures below show lines passing through a point. The measures of some of the angles are given in each. Calculate the other marked angles in each:



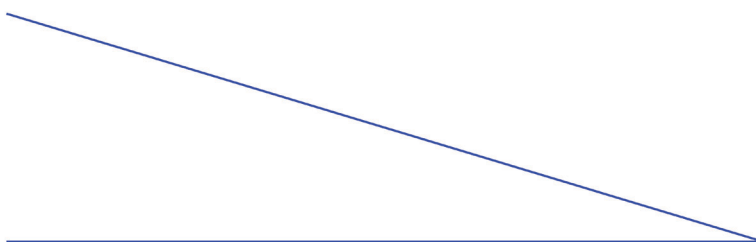
Non intersecting lines

See these lines:

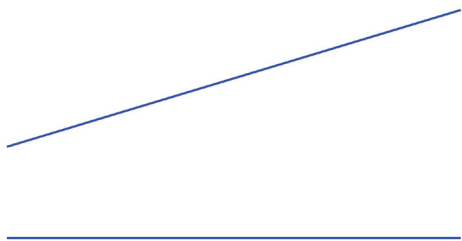


They don't meet

What if we extend them to the right?



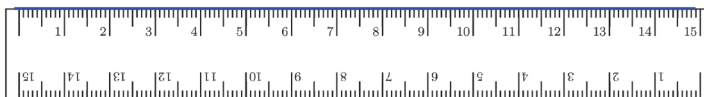
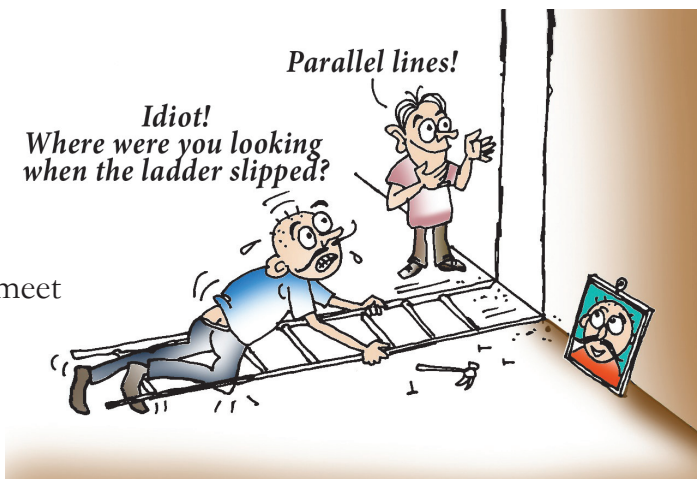
What about these lines?



How did you decide that they will meet if we extend them to the left?

Can we draw two lines which won't meet however much we extend them, in any direction?

Suppose we draw along the edges of a scale?



Will these lines meet if we extend them in either direction? How can we be sure?

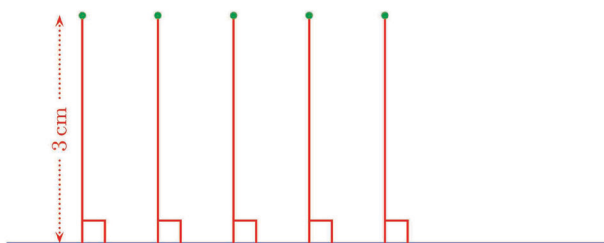


Lines like these, which keep the same distance apart and will never meet are called *parallel lines*.

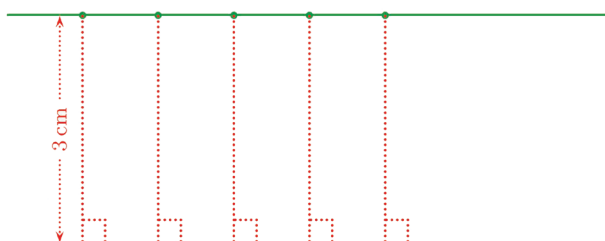
We can draw parallel lines along the edges of a scale. What if we want to increase the distance between them?

For example, suppose we want the distance between the lines to be 3 centimetres.

Draw some lines perpendicular to the first, all with height 3 centimetres:

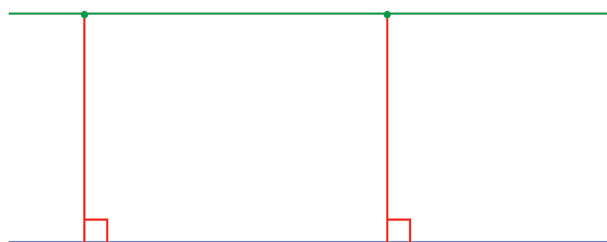


We just need to join their top ends:



Why so many perpendiculars?

Won't just two perpendiculars do?



Can we do this by drawing just one perpendicular?

But then the parallel we try to draw may turn out to be slanted:

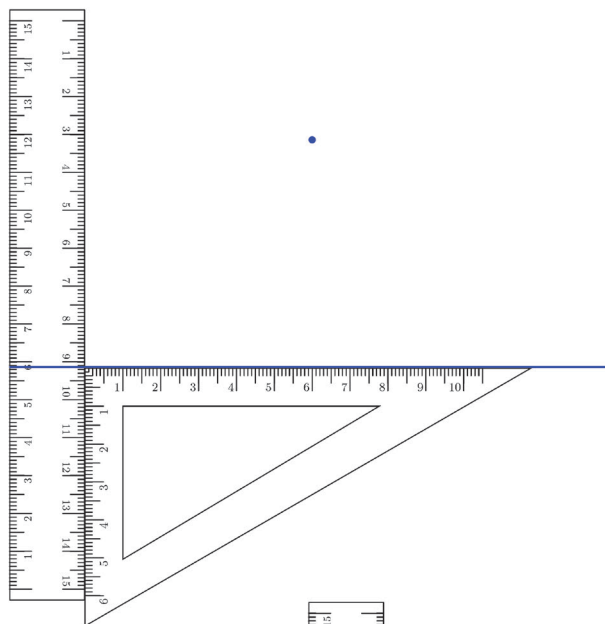


Have you found out how to draw?

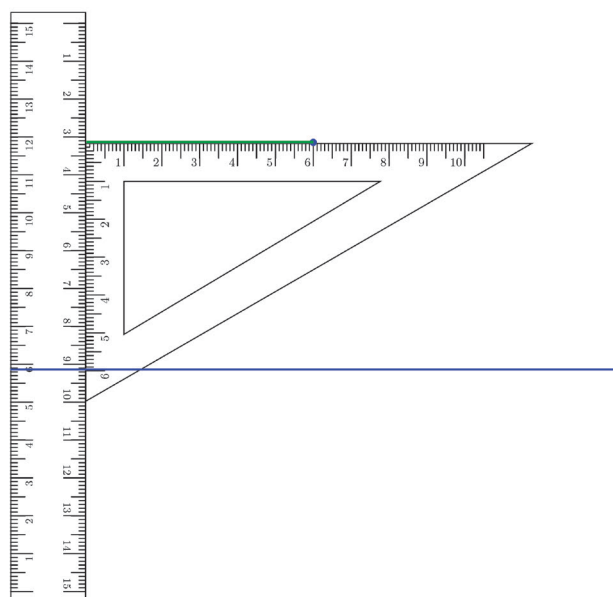


Can we do without actually drawing a perpendicular?

How about a scale kept perpendicular to the line?



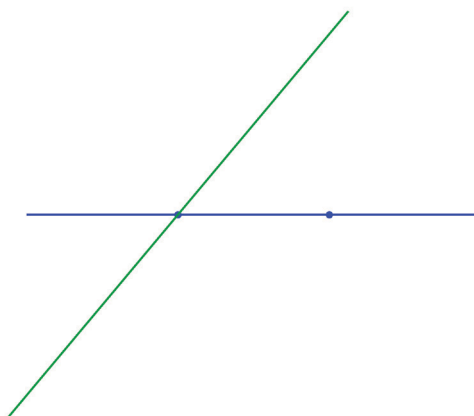
Now we can draw a parallel line, keeping the scale firmly in place and sliding the set square to the desired spot:



Draw a horizontal line in your notebook and draw a line parallel to it at a distance of 4 centimetres from it, using this technique

Line and slant

See this picture:



We want to draw a line parallel to the slanted line, through the point marked on the horizontal line

We can do it as before, using a scale and a set square.

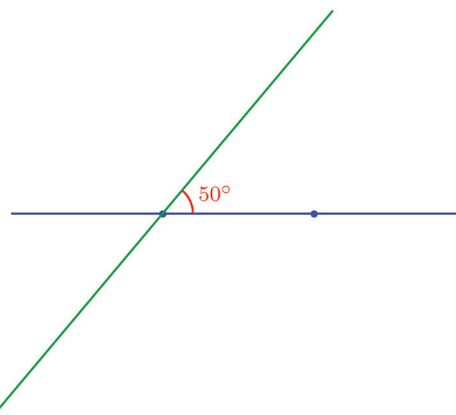
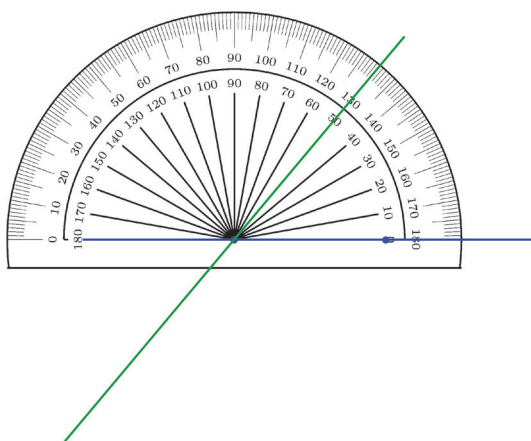
But it's a bit difficult to draw it slanted, right?

Any other way?

What we need is a slanted line through the point on the right.

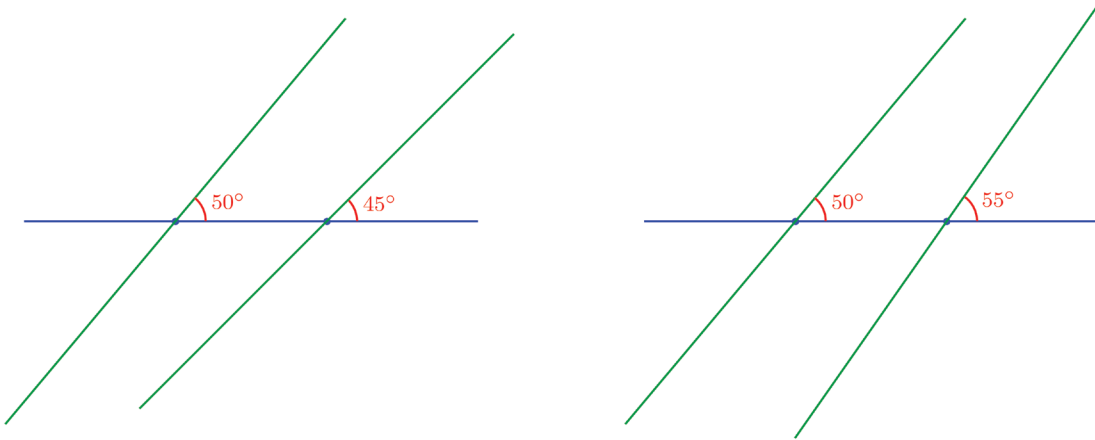
The question is how much slanted it should be.

Let's check how much the line in the picture is slanted:



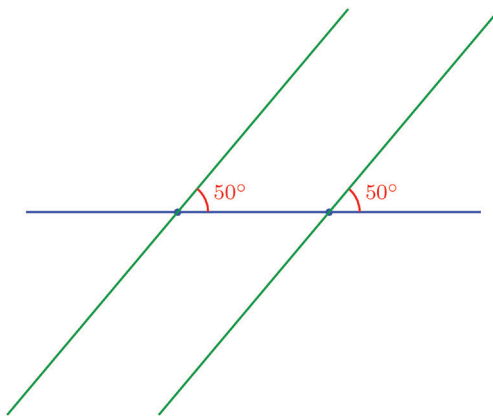
So how much slanted should be the parallel line we want to draw?

See these pictures:

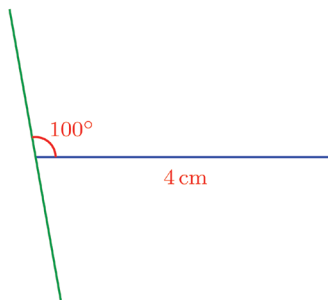


Are the lines parallel?

If they are to be parallel, what should be the angle on the right?

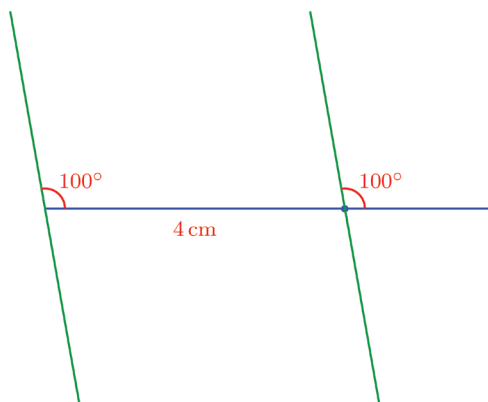


Now draw this picture in your notebook:



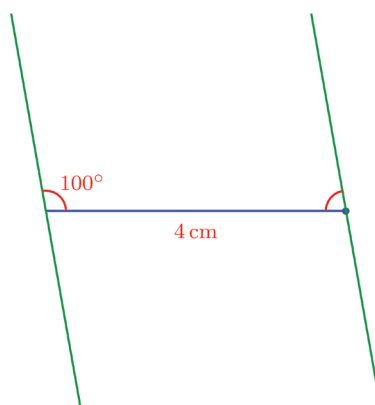
How to draw a line parallel to the slanted line through the right end of the horizontal line?

We can extend the horizontal line a bit to the right and draw the parallel line like this:



What should be the angle on the right?

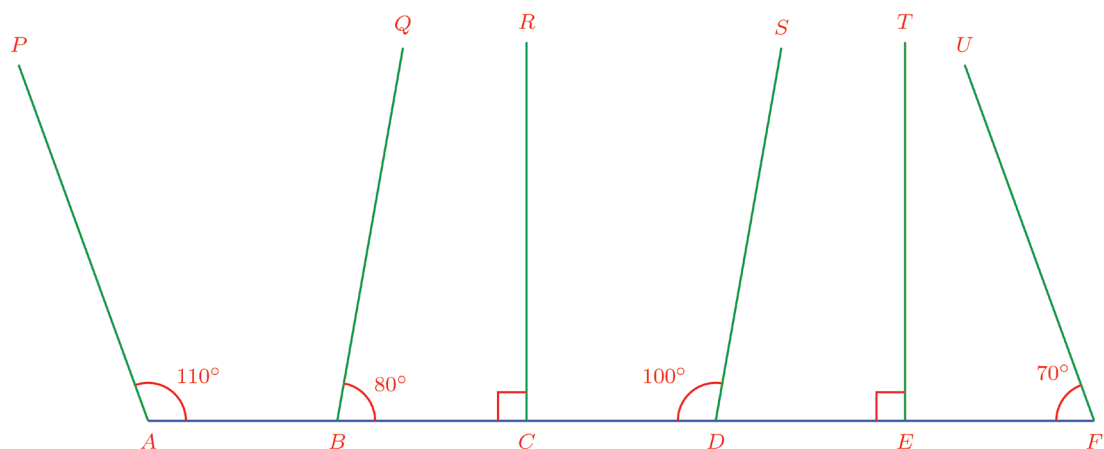
We can draw the parallel like this, without extending the horizontal line.



What should be the angle on the right?

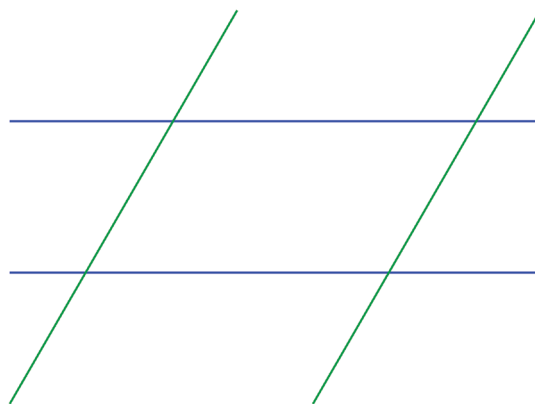


Find the pairs of parallel lines in the picture below and write them down:



Parallelograms

See this picture:



In this, the blue horizontal lines and the green slanted lines are two pairs of parallel lines. If we erase the parts beyond the intersecting points, we get a four-sided figure



Generally, any four-sided figure is called a *quadrilateral*.

For example, all figures shown below are quadrilaterals:

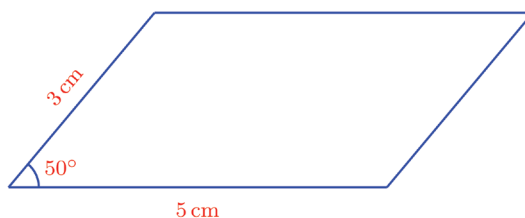


A quadrilateral with each pair of opposite sides parallel is called a *parallelogram*.

So the first quadrilateral we drew (formed by the intersection of two pairs of parallel lines) is a parallelogram.

Remember drawing such figures in Class 5, using scale and set squares? (the section **Line math** of the lesson **Lines and Circles**)

Can you draw this parallelogram in your note book using a protractor instead of a set square?



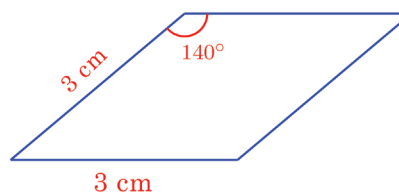
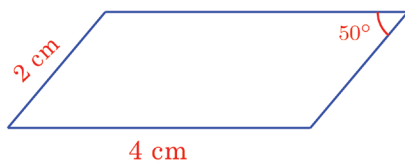
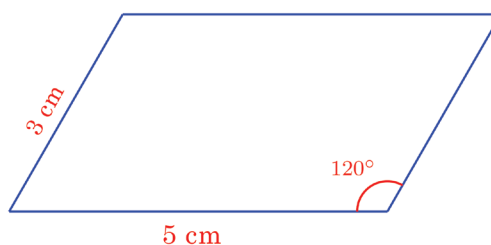
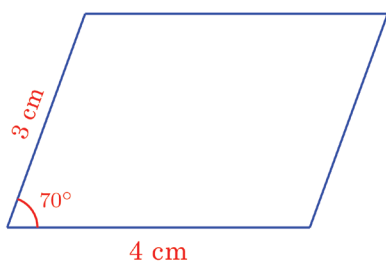
In what all ways can you draw it?

Draw other parallelograms with different lengths and angles.

If the angle between the lines is a right angle, what would the parallelogram become?



Draw the parallelograms shown below:



NUMBER RELATIONS



Digit sum

Every two-digit number is got by multiplying a number by 10 and then adding a number, isn't it?

For example,

$$20 = (10 \times 2) + 0$$

$$47 = (10 \times 4) + 7$$

$$98 = (10 \times 9) + 8$$

In all these, the multiplier of 10 is the first digit of the number and the number added is the second digit.

Suppose we write 10 as $9 + 1$ in these?

If we take 20

$$\begin{aligned} 20 &= 10 \times 2 \\ &= (9 + 1) \times 2 \end{aligned}$$

In this, $(9 + 1) \times 2$ can be split like this:

$$(9 + 1) \times 2 = (9 \times 2) + (1 \times 2)$$

(We have seen in the section **Addition, subtraction and multiplication** of the lesson **Number Relations** in Class 5 that to multiply a sum, we can multiply each number in the sum and add)

So, we get

$$\begin{aligned} 20 &= (9 + 1) \times 2 = (9 \times 2) + (1 \times 2) \\ &= (9 \times 2) + 2 \end{aligned}$$



Like this,

$$\begin{aligned} 30 &= 10 \times 3 \\ &= (9 + 1) \times 3 \\ &= (9 \times 3) + (1 \times 3) = (9 \times 3) + 3 \end{aligned}$$

Try writing 40, 50 and so on like this. Starting from the first, don't we get a pattern like this?

$$\begin{array}{ll} 10 = (9 \times 1) + 1 & \\ 20 = (9 \times 2) + 2 & 60 = (9 \times 6) + 6 \\ 30 = (9 \times 3) + 3 & 70 = (9 \times 7) + 7 \\ 40 = (9 \times 4) + 4 & 80 = (9 \times 8) + 8 \\ 50 = (9 \times 5) + 5 & 90 = (9 \times 9) + 9 \end{array}$$

In this, we wrote $90 = (9 \times 9) + 9$, instead of $90 = 9 \times 10$, to match the pattern. Now let's see how other two-digit numbers can be written, using this pattern. We can write

$$\begin{aligned} 11 &= 10 + 1 \\ &= (9 \times 1) + 1 + 1 \\ &= (9 \times 1) + 2 \end{aligned}$$

Like this, we can write

$$21 = 20 + 1 = (9 \times 2) + 2 + 1 = (9 \times 2) + 3$$

Thus, we get many patterns:

$$\begin{array}{llllll} 10 = (9 \times 1) + 1 & 20 = (9 \times 2) + 2 & 30 = (9 \times 3) + 3 & \dots\dots\dots & 90 = (9 \times 9) + 9 \\ 11 = (9 \times 1) + 2 & 21 = (9 \times 2) + 3 & 31 = (9 \times 3) + 4 & \dots\dots\dots & 91 = (9 \times 9) + 10 \\ 12 = (9 \times 1) + 3 & 22 = (9 \times 2) + 4 & 32 = (9 \times 3) + 5 & \dots\dots\dots & 92 = (9 \times 9) + 11 \\ \dots\dots\dots & \dots\dots\dots & \dots\dots\dots & \dots\dots\dots & \dots\dots\dots \\ 19 = (9 \times 1) + 10 & 29 = (9 \times 2) + 11 & 39 = (9 \times 3) + 12 & \dots\dots\dots & 99 = (9 \times 9) + 18 \end{array}$$

What do we see in all these?

Any two-digit number can be written as the sum of a multiple of 9 and a number

The multiplier of 9 is the first digit of the two-digit number. What about the number added?

For example, we wrote 32 like this:

$$32 = (9 \times 3) + 5$$

In this, how did we get the number 5 which is added?

$$32 = 30 + 2$$

$$30 = (9 \times 3) + 3$$

$$32 = (9 \times 3) + 3 + 2$$

So, what can we say about splitting any two-digit number like this?

Any two digit number is got by multiplying the first digit by nine and adding the sum of the two digits.

Let's look at three-digit numbers now. First let's split numbers like 100, 200, 300 and so on, as the sum of a multiple of 9 and a number

$$\begin{aligned} 100 &= 99 + 1 \\ &= (9 \times 11) + 1 \end{aligned}$$

We can write 200 like this:

$$\begin{aligned} 200 &= 100 \times 2 \\ &= (99 + 1) \times 2 \\ &= (99 \times 2) + (1 \times 2) = (99 \times 2) + 2 \\ &= (9 \times 11 \times 2) + 2 \\ &= (9 \times 22) + 2 \end{aligned}$$

What about 300?

$$\begin{aligned} 300 &= (99 + 1) \times 3 \\ &= (99 \times 3) + 3 \\ &= (9 \times 11 \times 3) + 3 \\ &= (9 \times 33) + 3 \end{aligned}$$

Try writing other numbers like this:

$$100 = (9 \times 11) + 1$$

$$200 = (9 \times 22) + 2$$

$$300 = (9 \times 33) + 3$$

$$400 = (9 \times 44) + 4$$

$$500 = (9 \times 55) + 5$$

$$600 = (9 \times 66) + 6$$

$$700 = (9 \times 77) + 7$$

$$800 = (9 \times 88) + 8$$

$$900 = (9 \times 99) + 9$$

How do we split 250 like this?

First we write 200 as

$$200 = (9 \times 22) + 2$$

Then we split 50 as done earlier:

$$50 = (9 \times 5) + 5$$

Now add these two. For that, we can first add the two multiples of 9 to make a single multiple of 9:

$$\begin{aligned}(9 \times 22) + (9 \times 5) &= 9 \times (22 + 5) \\ &= (9 \times 27)\end{aligned}$$

Thus we get

$$\begin{aligned}250 &= (9 \times 27) + 2 + 5 \\ &= (9 \times 27) + 7\end{aligned}$$

Here the multiplier of 9 is not the first digit of the three-digit number; but the number added is again the sum of the digits, right?

As another example, let's take 473. We can split 400 and 73 separately as done before:

$$400 = (9 \times 44) + 4$$

$$73 = (9 \times 7) + 7 + 3$$

In adding these two, we can combine the two multiples of 9 into a single multiple of 9:

$$\begin{aligned}473 &= 400 + 73 \\ &= (9 \times (44 + 7)) + 4 + 7 + 3 \\ &= (9 \times 51) + 14\end{aligned}$$

Here also, the number added is the sum of the digits.

Like this, any three-digit number we can write as a number added to a multiple of 9; and the number added is the sum of the digits of the three-digit number.

Next, what about a four-digit number?

First we write.

$$\begin{aligned} 1000 &= 999 + 1 \\ &= (9 \times 111) + 1 \end{aligned}$$

and

$$\begin{aligned} 2000 &= 1000 \times 2 \\ &= (999 + 1) \times 2 \\ &= (999 \times 2) + 2 \\ &= (9 \times 111 \times 2) + 2 \\ &= (9 \times 222) + 2 \end{aligned}$$

Continuing we get

$$\begin{aligned} 1000 &= (9 \times 111) + 1 \\ 2000 &= (9 \times 222) + 2 & 6000 &= (9 \times 666) + 6 \\ 3000 &= (9 \times 333) + 3 & 7000 &= (9 \times 777) + 7 \\ 4000 &= (9 \times 444) + 4 & 8000 &= (9 \times 888) + 8 \\ 5000 &= (9 \times 555) + 5 & 9000 &= (9 \times 999) + 9 \end{aligned}$$

Now any four-digit number can be split as before. For example let's take 4827. We split 4000 first, then 800 and finally 27 like this:

$$\begin{aligned} 4000 &= (9 \times 444) + 4 \\ 800 &= (9 \times 88) + 8 \\ 27 &= (9 \times 2) + 2 + 7 \end{aligned}$$

In adding all these together, we first combine all the multiples of 9 into a single multiple of 9 as before. That is,

$$\begin{aligned} 4827 &= (9 \times (444 + 88 + 2)) + 4 + 8 + 2 + 7 \\ &= (\text{Multiple of } 9) + (4 + 8 + 2 + 7) \end{aligned}$$

We can split any number like this, whatever be the number of digits

What can we say in general?

Any natural number is equal to the sum of a multiple of nine and the sum of the digits of the number

Let's look at a use of this result

As seen earlier, it's quite easy to split a two-digit number like this. For example,

$$35 = (9 \times 3) + (3 + 5) = (9 \times 3) + 8$$

So, what is the remainder on dividing 35 by 9?

Now suppose we take 65 instead of 35?

$$65 = (9 \times 6) + (6 + 5) = (9 \times 6) + 11$$

The remainder on dividing 65 by 9 is not 11; there is a 9 in 11 also, right?

So, to compute the remainder, let's write 65 like this:

$$\begin{aligned} 65 &= (9 \times 6) + 11 \\ &= (9 \times 6) + 9 + 2 \\ &= (9 \times 7) + 2 \end{aligned}$$

From this, we get the remainder as 2

How do we find the remainder on dividing 876 by 9?

First we split 876 as a multiple of 9 added to the sum of digits

We need only the remainder; so we need not calculate what the multiple of 9 is. So, we can write like this:

$$876 = (\text{Multiple of } 9) + 8 + 7 + 6 = (\text{Multiple of } 9) + 21$$

To calculate the remainder, we divide 21 by 9 and write like this:

$$21 = (9 \times 2) + 3$$

Now when we add the multiple of 9 in 876 and the multiple of 9 in 21, we get another multiple of 9. Thus 876 is 3 added to a multiple of 9; in other words, 3 is the remainder on dividing 876 by 9.

Here we got 3 by dividing 21 by 9; how did we get 21?

Like this, how can we calculate the remainder on dividing 5798 by 9?

5798 can be written as a multiple of 9 added to the sum of the digits $5 + 9 + 7 + 8 = 29$:

$$5798 = (\text{Multiple of } 9) + 29$$

Now we divide 29 by 9 and write

$$29 = (9 \times 3) + 2$$

Thus we can write 5798 as a multiple of 9 added to 2 (how?)

So, what is the remainder on dividing 5798 by 9?

We can compute the remainder on dividing any number by 9 like this

The remainder on dividing any number by 9 is the remainder on dividing the sum of the digits of the number by 9

What if the sum of the digits is a multiple of 9?

Then the remainder on dividing the sum of digits by 9 is 0; according to the general result above, the remainder on dividing the number itself by 9 also is 0. In other words, the number is a multiple of 9

If the sum of the digits of a number is a multiple of 9, then the number itself is a multiple of 9

We can see another thing from these.

We wrote a number as a multiple of 9 added to the sum of the digits of the number to calculate the remainder on dividing the number by 9

For example

$$356 = (9 \times 38) + 3 + 5 + 6 = (9 \times 38) + 14$$

The multiple of 9 here is a multiple of 3 also:

$$356 = (3 \times 3 \times 38) + 14 = (3 \times 114) + 14$$

So, what is the remainder on dividing 356 by 3?

To calculate it, we have to divide 14 by 3:

$$14 = (3 \times 4) + 2$$

Using this, we can write 356 like this:

$$356 = (3 \times 114) + (3 \times 4) + 2 = (3 \times 118) + 2$$

Thus the remainder on dividing 356 by 3 is 2

In general how do we calculate the remainder on dividing a number by 3?

The remainder on dividing any number by 3 is the remainder on dividing the sum of the digits of the number by 3

As a special case of this, we also get this:

If the sum of the digits of a number is a multiple of 3, then the number itself is a multiple of 3



(1) Complete the table below:

Number	Sum of the digits	Remainder on division by 9	Remainder on division by 3	Multiple of 9 Yes/No	Multiple of 3 Yes/No
764	17	8	2	No	No
954					
582					
4808					

- (2) The first three digits of a four-digit number, which is a multiple of 9, are 2, 5, 7. What is the number?
- (3) (i) Any number, and the number got by changing the order of its digits in any way, both leave the same remainder on division by 9. Why?
- (ii) If we change the order of the digits of a multiple of 9 in any way, will we again get a multiple of 9. Why?
- (4) Suppose we take all five-digit numbers made using the digits 1, 2, 3, 4, 5 without repetition
- (i) Which of them are multiples of 3?
- (ii) Which of them are multiples of 9?
- (iii) What is the remainder on dividing any of these numbers by 9?
- (5) Among all the ten-digit numbers using all of the ten digits 0, 1, 2, . . . , 9, how many are primes? What is the reason?
- (6) Which is the least four-digit number without any digit repeating, which is a multiple of 9? What about the least five-digit number?

Playing with nine

We have seen that any number is a multiple of 9 added to the sum of the digits of the number

So, what if we subtract from a number, the sum of its digits?

If we subtract from a number the sum of its digits, we get a multiple of 9

Is there any relation between the digits of a two-digit number and the number got by subtracting the sum of its digits?

Let's look at another problem. Take two numbers with the same digit-sum. For example, 51 and 24

We can split them like this:

$$51 = (9 \times 5) + 6$$

$$24 = (9 \times 2) + 6$$

So, how do we write $51 - 24$?

This difference is the same as the difference of the numbers got by subtracting 6 from each (the section **Unchanging differences** of the lesson **Unchanging Relations**)

That is

$$51 - 24 = (9 \times 5) - (9 \times 2)$$

Next, how do we write this difference of two multiples of 9 as a multiple of 9?

$$51 - 24 = (9 \times 5) - (9 \times 2)$$

$$= 9 \times (5 - 2)$$

$$= 9 \times 3$$

Can't we write the difference of any two numbers with the same digit-sum as the difference of two multiples of 9? So, this difference is a multiple of 9.

If the sum of the digits of two numbers are equal, then their difference is a multiple of 9

If we interchange the digits of a two-digit number, their digit-sums don't change, right?

For example if we interchange the digits of 73, we get 37 and the digit-sum of both is 10

So, by what we have noted above, their difference is a multiple of 9. Which multiple?

To calculate that, we write the numbers like this:

$$73 = (9 \times 7) + 10$$

$$37 = (9 \times 3) + 10$$

Then we can write

$$\begin{aligned} 73 - 37 &= (9 \times 7) - (9 \times 3) \\ &= 9 \times (7 - 3) \\ &= 9 \times 4 \end{aligned}$$

In this, $4 = 7 - 3$ is the difference of the digits, isn't it?

Like this, we can see that if we take any two-digit number, write it in reverse and subtract the smaller from the larger, we get the difference of their digits multiplied by 9.

From this, we get this pattern:

$21 - 12 = 9$	$31 - 13 = 18$	
$32 - 23 = 9$	$42 - 24 = 18$	$41 - 14 = 27$
$43 - 34 = 9$	$53 - 35 = 18$	$52 - 25 = 27$
$54 - 45 = 9$	$64 - 46 = 18$	$63 - 36 = 27$
$65 - 56 = 9$	$75 - 57 = 18$	$74 - 47 = 27$
$76 - 67 = 9$	$86 - 68 = 18$	$85 - 58 = 27$
$87 - 78 = 9$	$97 - 79 = 18$	$96 - 69 = 27$
$98 - 89 = 9$		 $91 - 19 = 72$



- (1) The difference of any number and the number got by rearranging its digits in any manner is a multiple of 9. Why?
- (2) The difference of any three-digit number and the number written in reverse is a multiple of 99. Why? What is the relation between the number by which 99 is multiplied and the digits of the original number?
- (3) What is the relation between the digits of the number got by multiplying 99 by a single-digit number?

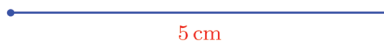
4. Take any three-digit number, write it in reverse and find the difference of these two numbers. Write the number got as difference in reverse and add to the difference. What number do you get? Do this with other three-digit numbers. Do you get 1089 always? For what type of numbers do you get a different answer?

LETTER MATH

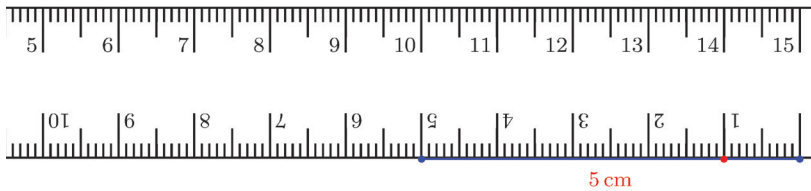


Addition and subtraction

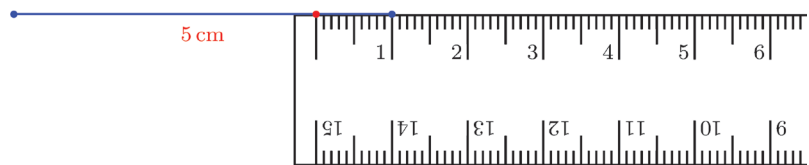
A simple problem. First draw a 5 centimetre long line:



Next mark a point on it, 1 centimetre from the right end. Did you put the scale like this?

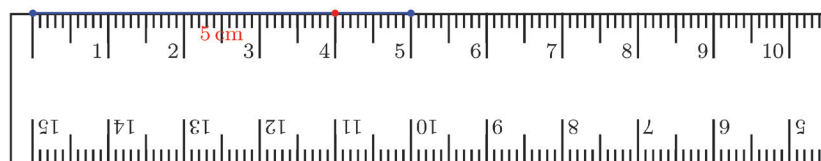


Or like this?



Any other way to place the scale?

How about this?



What did we do here?

Instead of marking the point 1 centimetre from the right end, we marked it 4 centimetres from the left end.

Why is this correct?

What is the relation between the distances of the point from the left end and its distance from the right end?

Subtracting the distance from the right end from 5 centimetres gives the distance from the left end.

We can shorten this a bit like this:

5 centimetres – distance from the right end = distance from the left end.

There's a way to shorten this further.

If we write the number denoting the distance from the right end as r and the number denoting the distance from the left end as l , then we can write the relation between the distances like this:

$$5 - r = l$$

In this, as we change the position of the marked point, both l and r change; but the relation above doesn't change, does it?

For example, if we want to mark a point 2 centimetres from the right, at what distance from the left end should we mark it?

$$5 - 2 = 3 \text{ cm}$$

We can say it like this:

If we take $r = 2$ in the relation $5 - r = l$, then $l = 5 - 2 = 3$

In this relation, l and r can be any numbers between 0 and 5; but the number r subtracted from 5 should be the number l .

We can say it in another way: if we add the numbers l and r , we get 5. That is,

$$l + r = 5$$

What does this mean?

The distance from the left and the distance from the right added together gives 5 centimetres:



Is there any other way to say the relation between the left and right distances?

Now suppose we draw the first line 6 centimetres long instead?

If we denote the left and right distances by l and r , as in the first problem, how do we write the relation between them?

What can we say in general?

In any line if we mark a point between its ends, then the sum of its distances from the end points is the length of the line.

Here the length of the line is the distance between its end points, right? So we can also say it like this:

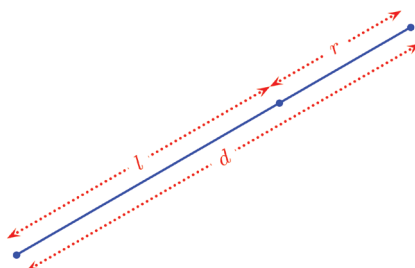
If three points are on the same straight line, then the distance between the points at the two ends is equal to the sum of the distances from the end points to the point in between.

How do we say this using letters?

We can denote the distances from the end points to the point in between by l and r as before; and the distance between the end points we can denote by d .

Then we can write the relation between these distances as

$$d = l + r$$



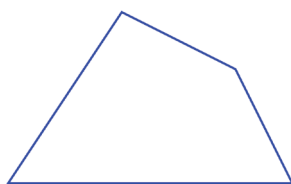
In this, we can take l , r and d as any three numbers; but the sum of l and r should be d

We can also write this relation in two other ways:

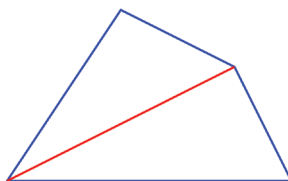
$$l = d - r$$

$$r = d - l$$

Let's look at another problem. See this picture:



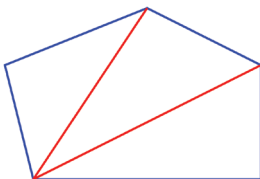
A quadrilateral. If we draw a line from one corner to the opposite corner, we can split it into two triangles:



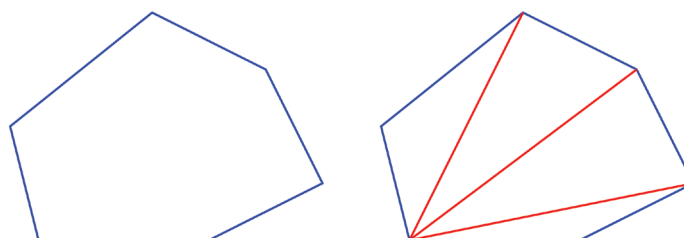
What about a figure with five sides?



From a corner we can draw two lines within to split this into three triangles



And in a figure with six sides?

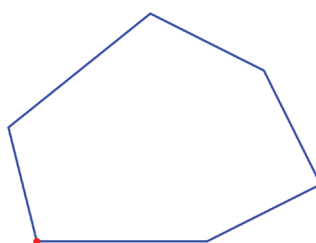


Like this, draw figures of seven and eight sides and draw lines within from one corner to split them into triangles. Also, make a table like the one below:

Sides	Lines within	Triangles
4	1	2
5	2	3
6	3	4
7		
8		

Figures like these are called *polygons* in general. Do you see any relation between the number of sides of a polygon and the number of lines we can draw within the polygon from one corner to all other corners?

Let's look at the picture of the six-sided polygon drawn earlier:



From the bottom left corner, to which other corners can we draw lines?

We have already drawn two sides joining this corner and the corners next to it on either side.

Apart from these two corners and the corner we start from, how many other corners are there?

Now in any polygon, once we choose a corner, then excluding this and the corner right next to it on either side, we can draw lines by joining this corner to all other corners

Thus we see that

$$\text{Number of lines} = \text{Number of corners} - 3$$

In any polygon, the number of corners is the same as the number of sides, isn't it?

So, we can say

$$\text{Number of lines} = \text{Number of sides} - 3$$

This relation also we can write using letters as in the distance problem.

Writing s for the number of sides of the polygon and l for the number of lines that can be drawn within the polygon joining a fixed corner to others, we can write

$$l = s - 3$$

We can say this in another way. When we change the number of sides of the polygon, the number of lines we can draw like this also changes. But their difference is 3 for all polygons. Using letters,

$$s - l = 3$$

Like this, can you find the relation between the number of sides of a polygon and the triangles formed within it by such lines?

If we write the number of triangles as t , in what all ways can we write the relations between s and t ?

What about the relation between l and t ?

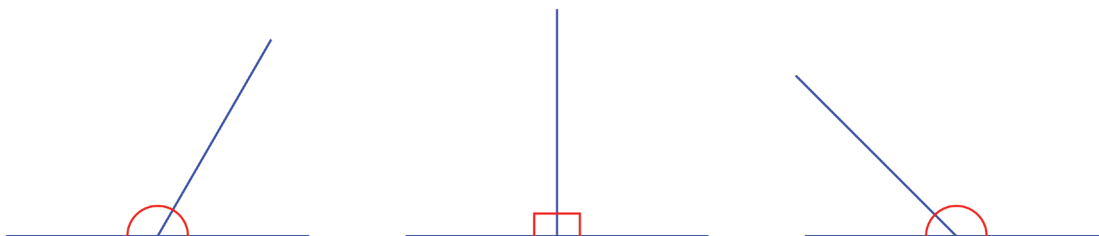
In this, s can be any natural number greater than 3. What about l and t ?

But these numbers should have these relations between them.

$$l = s - 3, \quad t = s - 2$$



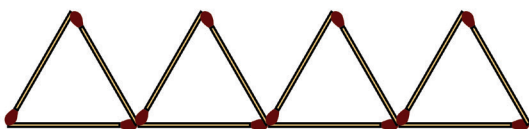
(1) The pictures below show a line drawn from a point on another line:



- (i) In what all ways can we say the relation between the angles marked on the left and right of the top line?
 - (ii) If we denote the measure of the angle on the left by l and the measure of the angle on the right by r , then in what all ways can we write the relation between the numbers l and r ?
 - (iii) In all cases, l and r are between what numbers?
- (2)
- (i) Each side of a square of perimeter 40 centimetres is extended by 1 centimetre to make a larger square. By how much is the perimeter increased?
 - (ii) If the sides of a square of perimeter 60 centimetres are extended like this, what would be the perimeter of the large square?
 - (iii) Is the difference in perimeters of any square, and the larger square made by extending each side by 1 centimetre the same number for all squares? Why?
 - (iv) Is it also true for rectangles which are not squares?
 - (v) If the perimeter of the larger rectangle is denoted by p and the perimeter of the original small rectangle is denoted by s , in what all ways can we write the relation between these two numbers?

Letter multiplication

See this picture:



How many triangles are there in it?

How many matchsticks were used to make them?

How did you calculate?

Did you add $3 + 3 + 3 + 3 = 12$

Or did you multiply $4 \times 3 = 12$?

How many matchsticks do we need to make 10 such triangles?

In general, the number of matchsticks we need is the number of triangles we want to make multiplied by 3, right?



How about shortening this statement, using letters?

If we write the number of triangles as t and the number of matchsticks as m , then what is the relation between the numbers t and m ?

$$m = t \times 3$$

When we write letters to denote numbers, we usually don't write the multiplication sign. That is, we write $3 \times x$ as just $3x$, without the multiplication sign in between. (Maybe because the multiplication sign $\times$ and the letter x look a lot alike). Moreover, we write $x \times 3$ as $3x$.

So, we write the relation between the number t denoting the number of triangles and the number m denoting the number of matchsticks, like this:

$$m = 3t$$

This is the shorthand for the statement that the number of matchsticks is three times the number of triangles needed.

Here we can take any number as t ; and m is a multiple of 3, since it is three times t .

On the other hand, if we take some multiple of three, with that many matchsticks how many triangles can we make?

For example, with 45 matchsticks, how many triangles can we make?

$$45 \div 3 = 15$$

In general, the number of triangles is the number of matchsticks divided by 3.

How do we write this using letters?

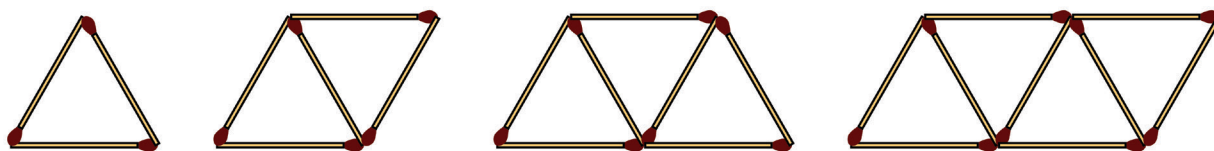
$$t = m \div 3$$

We have seen in class 5 that any division can be written as a fraction (the section **Fraction and division** of the lesson **Fractions**)

So, we can write the relation between the number of matchsticks and the number of triangles like this also:

$$t = \frac{m}{3}$$

Now suppose we make triangles like this?

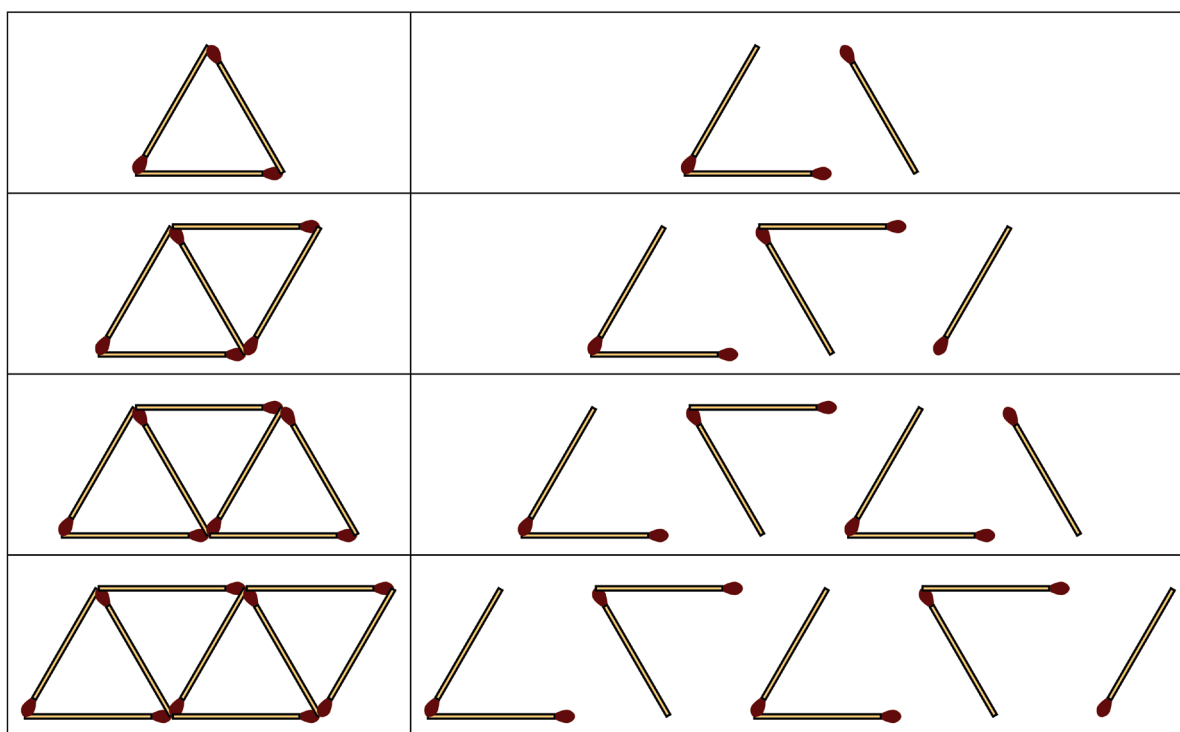


Write the number of triangles and the number of matchsticks as a table like this:

Triangles	Matchsticks
1	3
2	5
3	7
4	9
5	
6	

See any relation between the number of triangles and the number of matchsticks?

Look how the triangles are made in this way:



Then we can see the relation between the number of triangles and the number of matchsticks like this:

Triangles	Matchsticks
1	$2 + 1$
2	$(2 \times 2) + 1$
3	$(3 \times 2) + 1$
4	$(4 \times 2) + 1$
5	
6	

In general, the number of matchsticks used when triangles are made this way is one added to twice the number of triangles.

How do we write this in shorthand, using letters?

Writing the number of triangles as t and the number of matchsticks as m ,

$$m = (t \times 2) + 1$$

Here also, we write $t \times 2$ without the multiplication sign. Also, in writing the product of an actual number and a letter denoting a number, we write the actual number first, so that we write $t \times 2$ as $2t$.

Thus the above relation we write as

$$m = 2t + 1$$

Now to calculate how many matchsticks are needed to make 10 triangles like this, we need only take $t = 10$ in this relation. Then we get

$$m = (2 \times 10) + 1 = 21$$

Thus to make 10 triangles like this, we need 21 matchsticks.

Here the number of triangles can be as many as we want. This means t can be any natural number; and m is the number got by adding 1 to twice that number.

So, m is an odd number; and it should be three or more (why?).

Now if we take such a number, how many triangles can we make with that many matchsticks?

For example suppose we take 45 matchsticks.

We can divide 45 by 2 and write it as a multiple together with a remainder.

$$45 = (22 \times 2) + 1$$

If we think in reverse we see that to make 22 triangles we need $(22 \times 2) + 1 = 45$ matchsticks

Thus the number of triangle is the quotient 22 on dividing 45 by 2.

To find this quotient, we need only divide 44 by 2, right?

That is,

$$45 - 1 = 44$$

$$44 \div 2 = 22$$

In the same way, how do we calculate the number of triangles that can be made with 51 matchsticks?

$$51 - 1 = 50$$

$$50 \div 2 = 25$$

How many matchsticks are needed to make 25 triangles?

$$(2 \times 25) + 1 = 51$$

So, if we take an odd number greater than 3, how do we calculate the number of triangles we can make with that number of matchsticks?

(i) Subtract 1 from the number

(ii) Divide by 2

How do we write this using letters?

If m is an odd number greater than 3, then the number of triangles we can make, as we have been now doing, with m matchsticks can be calculated like this:

(i) 1 subtracted from m is $m - 1$

(ii) Dividing by 2 gives $(m - 1) \div 2$

Thus if m is an odd number greater than 3, the number of triangles we make in this fashion with m matchsticks is

$$t = \frac{m-1}{2}$$

Let's look at another thing

We have made matchstick triangles in two different ways

To make one triangle, in whichever way, we need 3 matchsticks.

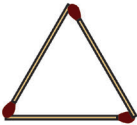
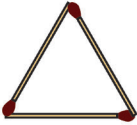
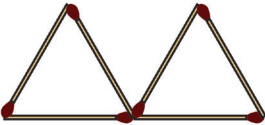
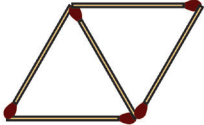
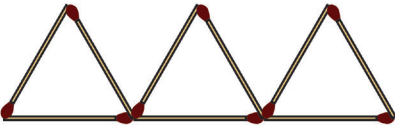
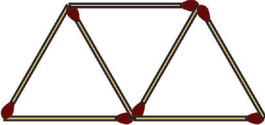
What about two triangles?

How many matchsticks do we need in the first way?

And in the second way?

What is the difference in the number of matchsticks?

See this table

Triangles	Method	Matchsticks	Difference
1		3	0
		3	
2		6	1
		5	
3		9	2
		7	

Did you notice the difference in the number of matchsticks needed, when we make the same number of triangles in two different ways?

If 4 triangles are made in each of these ways, how many more sticks would we need in the first way?

If we make 5 triangles?

Is there any relation between the number of triangles and the difference in the number of matchsticks needed?

If we denote the number of triangles by t and the difference in the number of matchsticks needed in the two ways by d , how do we write this relation?

$$d = t - 1$$

Why is this difference always one less than the number of triangles?

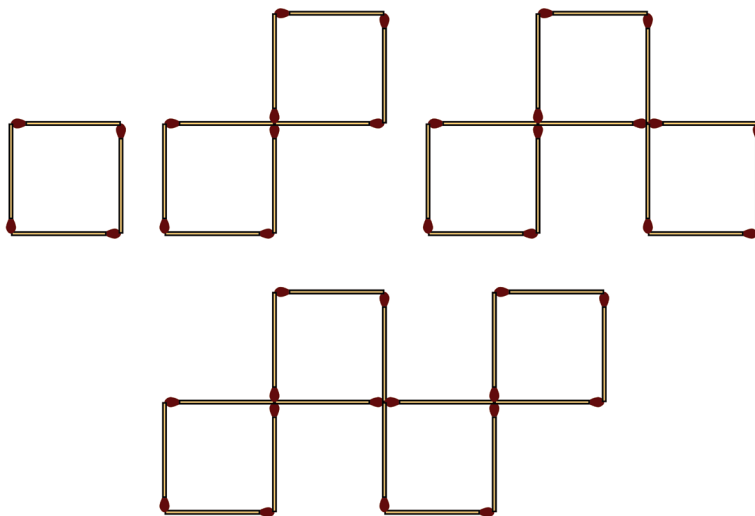
Now can you find the answer to this question?

One kid made 10 matchstick triangles in the first way and another kid made 10 triangles in the second way. Who used more matchsticks? How many more?

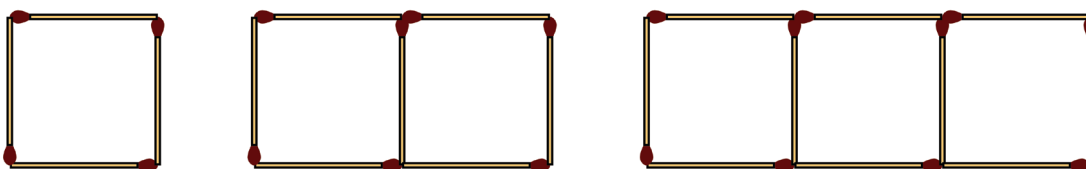
It is easy to say that the first kid used more matchsticks. To calculate how many more, do we need to calculate the number of matchsticks each used? What is the simpler way?



(1) We can make squares also with matchsticks:



- (i) As we go on making such squares, what is the relation between the number of squares and the number of matchsticks used?
 - (ii) Write this relation, denoting the number of squares by s and the number of matchsticks by m
 - (iii) What kind of numbers are s and m in this?
 - (iv) For a multiple of 4, how do we calculate the number of squares that can be made with that many matchsticks?
 - (v) Write this calculation also using letters
- (2) We can make matchstick squares like this also:



- (i) Write as a table the number of squares and the number of matchsticks needed to make squares like this and find the relation between these numbers.
- (ii) Explain this relation by drawing pictures showing how the squares are made.
- (iii) Write this relation denoting the number of squares by s and the number of matchsticks used by m .
- (iv) What kind of numbers are s and m in this?
- (v) If we take a number which leaves a remainder 1 on division by 3, how do we calculate the number of squares that can be made like this using that many matchsticks?
- (vi) Write this calculation also using letters.

- (3) (i) When equal number of matchstick squares are made in the two different ways as in the first and second question, what is the difference of the number of matchsticks used?
- (ii) Denoting the number of squares as s and the difference in the number of matchsticks used in the two ways as d , how do we write the relation between s and d .
- (iii) Explain the reason for this relation.
- (iv) One kid uses 25 matchsticks to make squares in the second way. How many matchsticks would be needed to make the same number of squares in the first way?

(4) See these pictures:



The first picture is a square of sides 1 centimetre. The second picture is a rectangle made by joining together two such squares. The third picture is a rectangle made by joining together three such squares.

- (i) Calculate the perimeter of each.
- (ii) What is the perimeter of the rectangle made by joining together four such squares like this? What about the rectangle made with five squares?
- (iii) Make a table of the number of squares used to make the rectangles and the perimeter of the rectangles, starting from the first. What is the relation between the number of squares and the perimeter? Explain the reason for this relation?
- (iv) Denoting the number of squares by s and the perimeter by p , write the relation between them.
- (v) To get a rectangle of perimeter 1 metre, how many squares must be joined like this?

DATA PICTURES



Line graphs

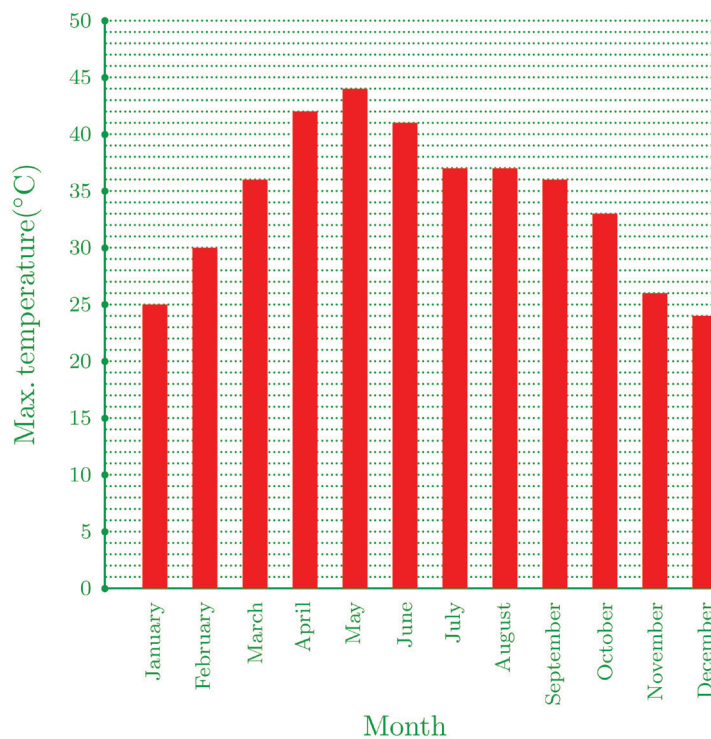
We have seen how different kinds of information in the form of numbers can be given as tables or pictures (The lesson **Tables and Graphs** in the class 5 textbook).

The table below shows the maximum temperature recorded in Delhi during various months of 2024.

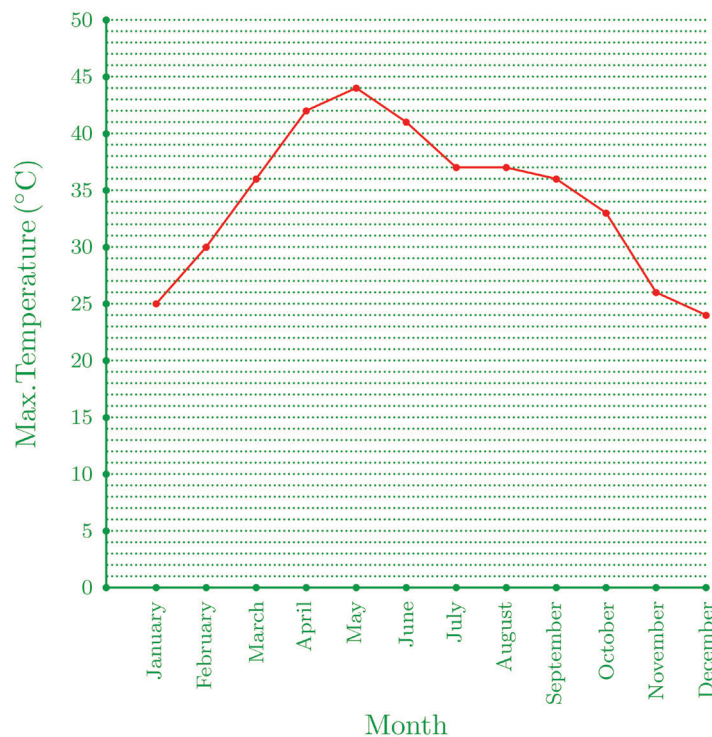
Month	Maximum temperature (°C)
January	25
February	30
March	36
April	42
May	44
June	41
July	37
August	37
September	36
October	33
November	26
December	24



We can show these as a picture, as we did in Class 5:



Usually pictures of measurements which change with respect to time are drawn in a different way:



How is this picture drawn?

As in the first picture, months are marked at equally spaced points along a horizontal line, and temperatures are marked at equally spaced points along a vertical line

Then instead of drawing rectangles, a single point is marked above each month, at the height of the maximum temperature in that month

Joining all these points above the months gives the above picture

Such a picture showing numerical data is called a *line graph* or *line chart*

The first kind of picture using rectangles is called a bar graph or bar chart (All the pictures we saw in the lesson **Tables and Graphs** in Class 5 are bar graphs)

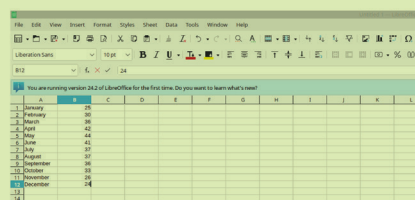
Now if we look at the line graph, we can tell at a glance that the maximum temperature increases steadily from January, reaches a peak in May, and then decreases during the following months.

Can you find these from the graph?

- What is the highest temperature of the year?
- In which month was it?
- Which of the months in the year can be called the summer season?

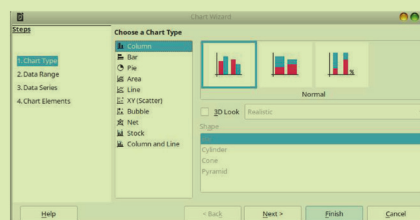
Computer picture 1

Nowadays we draw pictures of data using computers. We can do this using LibreOffice Calc, which is included in the KITE GNU-Linux. First let's draw the bar graph of the temperature data. For that we first type the data in the application:

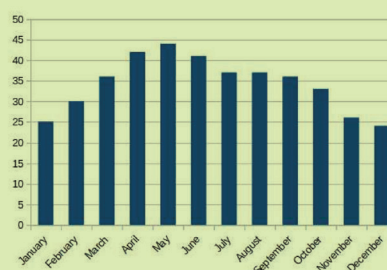


	A	B	C	D	E	F	G	H	I	J	K	L
1	January	24										
2	February	26										
3	March	36										
4	April	42										
5	May	44										
6	June	43										
7	July	37										
8	August	36										
9	September	35										
10	October	32										
11	November	26										
12	December	24										

Then we click on the  icon at the top right, which opens a window. In this we select Column as the Chart Type



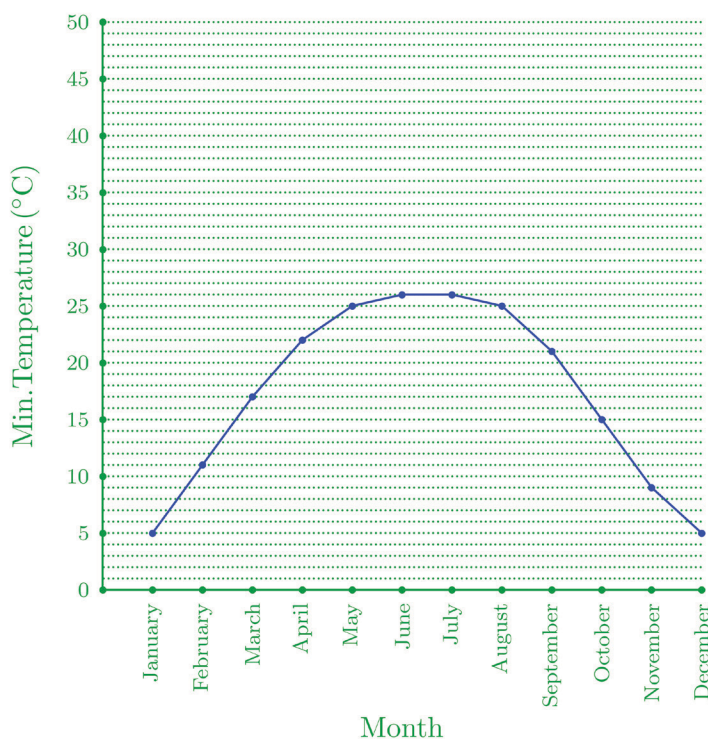
Finally, we click on the Finish button to get the graph



- Between which two consecutive months did the maximum temperature increase the most?
- And between which months did it decrease the most?

What other information do you get from the graph?

The line graph below shows the minimum temperatures recorded in Delhi during various months of the same year:

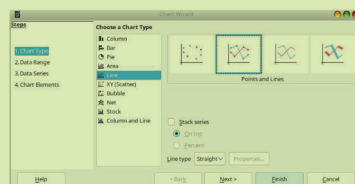


As in the case of the maximum temperatures, what all information can you find from this graph? Write them down.

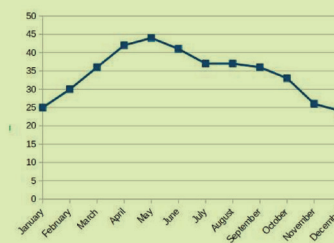
We have seen in Class 5 that when we want to compare two sets of data, we draw their bar graphs together. Here also, we can draw the line graphs of maximum temperature together in a single picture:

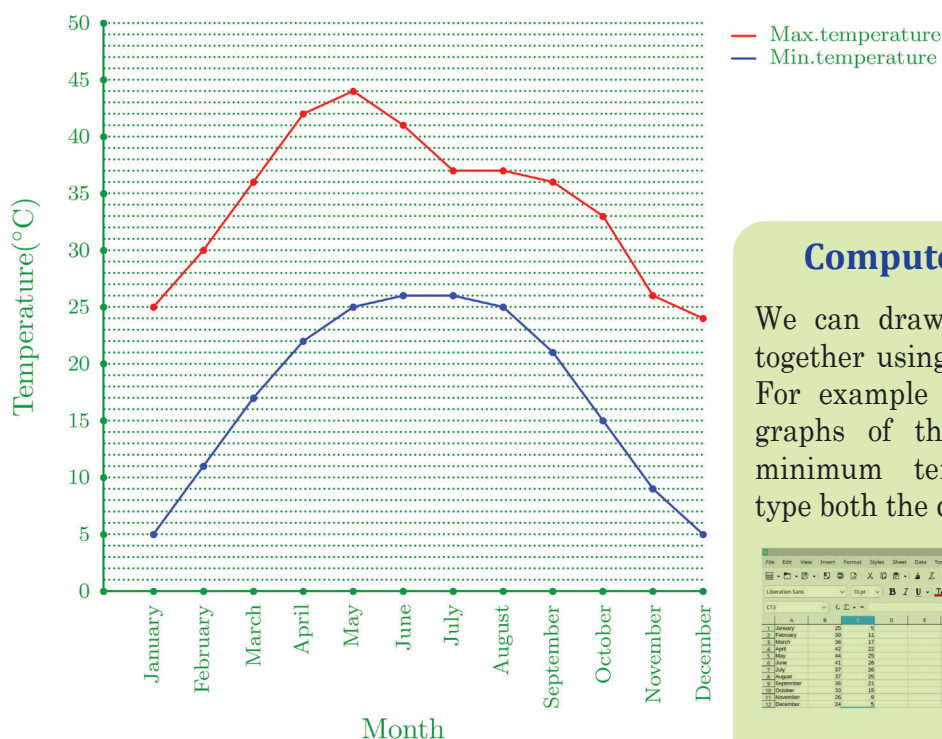
Computer picture 2

We can draw line graphs also using LibreOffice Calc. For that we first type the data in the application and then click on the  icon as before. In the window which opens up, we select Line and then click on. Points and Lines on the right side.



Finally we click on the Finish button to get the graph





Now can you answer the questions below just by looking at the graphs, without looking up the numbers?

- In which month is the difference in maximum and minimum temperatures the highest?
- In which month is this difference the least?
- During which two consecutive months does the maximum temperature remain the same?
- What about the minimum temperature?

What other things can you find out from the graphs alone?

Check whether your conclusions are correct by verifying the numbers.

Scores in a cricket match are also shown as line graphs.

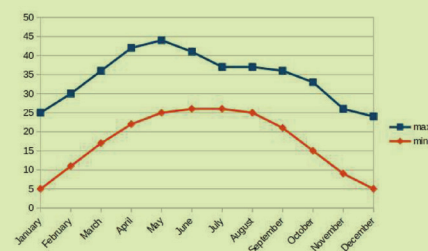
Computer picture 3

We can draw two line graphs together using LibreOffice Calc. For example to draw the line graphs of the maximum and minimum temperatures, first type both the data:

	A	B	C	D	E	F	G	H	I	J	K	L
1	January											
2	February											
3	March											
4	April											
5	May											
6	June											
7	July											
8	August											
9	September											
10	October											
11	November											
12	December											

Then as before, click on the icon, select Chart Type as Line in the window which opens up, and click on Points and Lines. The names for the different coloured graphs are shown as Column B and Column C. To change this, click on the Next button and then on Data Series. Now in Column B give max as Range for Name/ In the same way change Column C to min

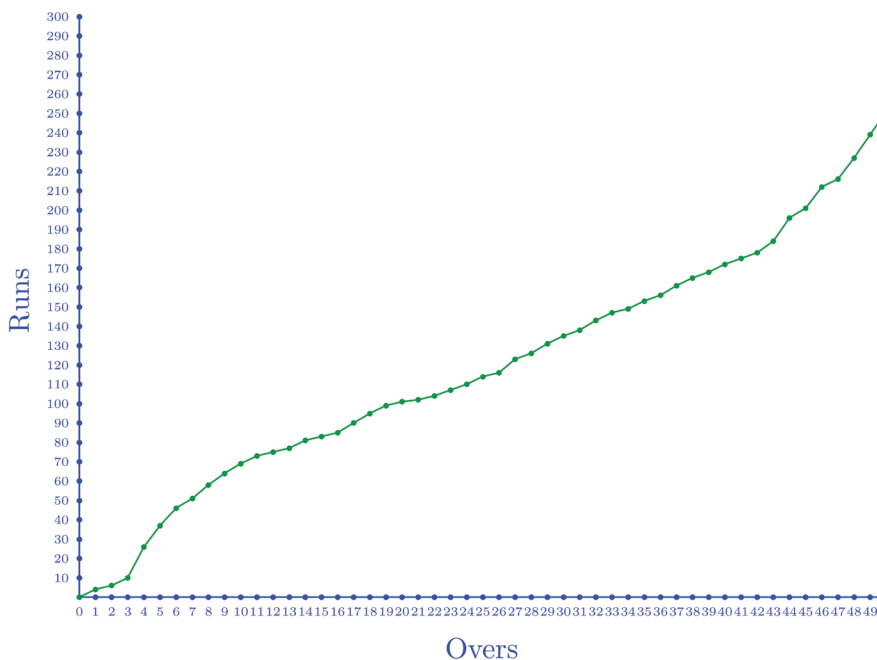
Clicking on the Finish button, we get this picture:



We have seen in Class 5 how the runs scored by a team in each over is shown as a bar graph.

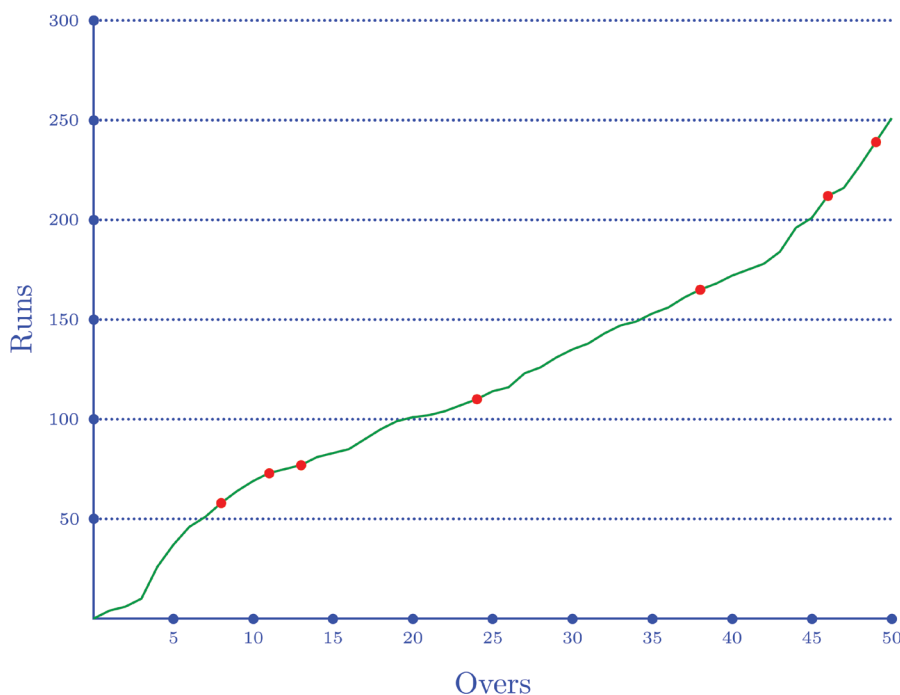
Instead of this, the total runs a team has scored after each over is often drawn as a line graph.

For example, here is the line graph of the runs scored by New Zealand, who batted first, in the 2025 ICC Champions trophy:

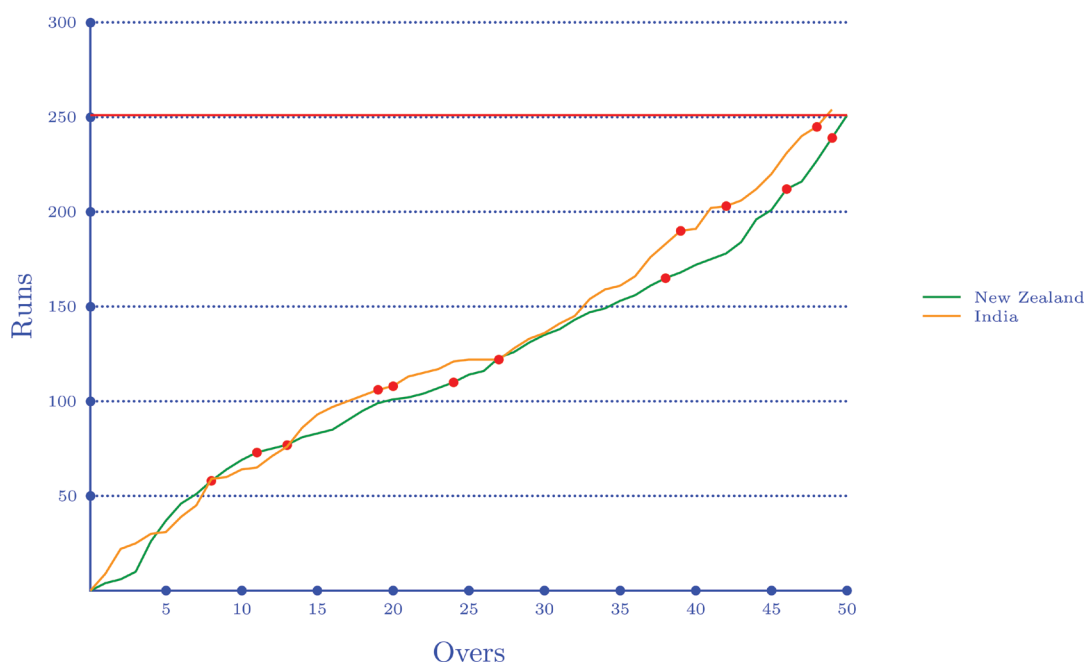


Usually in such graphs, overs are shown 5 apart and runs 50 apart; and the points showing total runs above the overs are not marked. Thus the picture above is usually drawn like this.

The red dots are marked above those overs in which a wicket was lost.



The line graph of the scores of India, which batted second, can also be shown together with this:



If we look closely at the right ends of these graphs, we can see that New Zealand scored a few runs more than 250 in 50 overs and India scored more runs before 50 overs and thus won the match.

Counting the red dots in each graph, we can see that New Zealand lost 7 wickets and India lost 6 wickets (The actual scores are New Zealand 251 for 7 in 50 overs and India, 254 for 6 wickets in 49 overs).

What all things can we get from the graphs?

- Which team scored more in the first 10 overs?
- Between which overs did the total score of India equal the total score of New Zealand at that stage?
- After this, did India's total score go below New Zealand's total score at any stage?

See what other details you can get from the graphs.



- (1) The graphs below show the maximum and minimum temperatures recorded during various months of 2023 in Palakkad district of Kerala

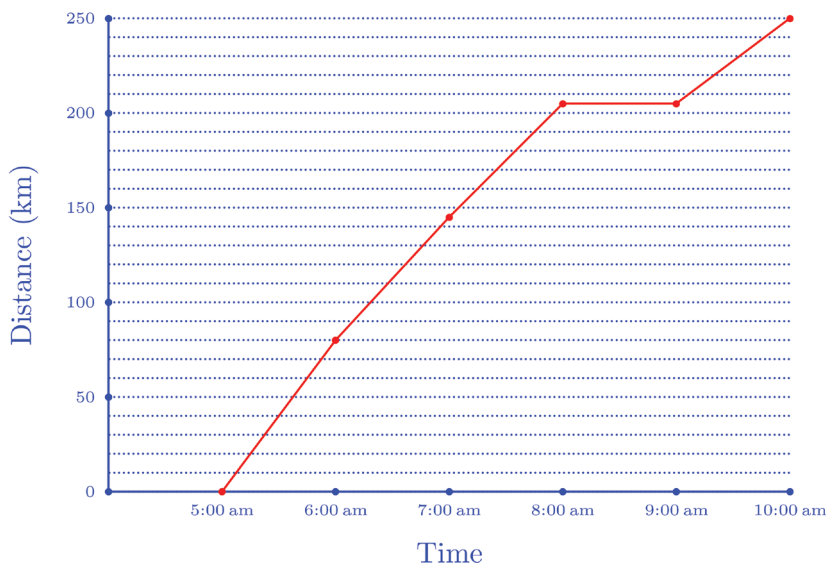


- In which month was the highest temperature recorded? How much was it?
- In which month was the lowest temperature recorded? How much was it?
- In which month was the difference of temperatures the highest? How much was it?
- Why was the temperature low during June–August?

- (2) The graph below shows the sale of two-wheelers in India from 2015 to 2024:



- (i) In which year was the most number of two-wheelers sold?
 - (ii) During which two years were the most number of two-wheelers sold?
 - (iii) Why was the sales very low during 2019–2022?
- (3) A person started a trip in car at five in the morning and reached his destination at ten. The graph below shows the total distance travelled at various points of time:



- (i) How much distance did he travel?
- (ii) How much time did the trip take?
- (iii) During which time did the car travel the most distance in one hour?
- (iv) How much distance did the car travel between six and seven?
- (v) And the distance travelled between eight and nine?
- (vi) During what time was the car stopped?

Notes

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]

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Notes

[illegible]

Notes

[illegible]

[illegible]

CONSTITUTION OF INDIA

Part IV A

FUNDAMENTAL DUTIES OF CITIZENS

ARTICLE 51 A

Fundamental Duties- It shall be the duty of every citizen of India:

- (a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;
- (b) to cherish and follow the noble ideals which inspired our national struggle for freedom;
- (c) to uphold and protect the sovereignty, unity and integrity of India;
- (d) to defend the country and render national service when called upon to do so;
- (e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;
- (f) to value and preserve the rich heritage of our composite culture;
- (g) to protect and improve the natural environment including forests, lakes, rivers, wild life and to have compassion for living creatures;
- (h) to develop the scientific temper, humanism and the spirit of inquiry and reform;
- (i) to safeguard public property and to abjure violence;
- (j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievements;
- (k) who is a parent or guardian to provide opportunities for education to his child or, as the case may be, ward between age of six and fourteen years.

CHILDREN'S RIGHTS

Dear Children,

*Wouldn't you like to know about your rights? Awareness about your rights will inspire and motivate you to ensure your protection and participation, thereby making social justice a reality. You may know that a commission for child rights is functioning in our state called the **Kerala State Commission for Protection of Child Rights**.*

Let's see what your rights are:

- Right to freedom of speech and expression.
- Right to life and liberty.
- Right to maximum survival and development.
- Right to be respected and accepted regardless of caste, creed and colour.
- Right to protection and care against physical, mental and sexual abuse.
- Right to participation.
- Protection from child labour and hazardous work.
- Protection against child marriage.
- Right to know one's culture and live accordingly.
- Protection against neglect.
- Right to free and compulsory education.
- Right to learn, rest and leisure.
- Right to parental and societal care, and protection.

Major Responsibilities

- Protect school and public facilities.
- Observe punctuality in learning and activities of the school.
- Accept and respect school authorities, teachers, parents and fellow students.
- Readiness to accept and respect others regardless of caste, creed or colour.



Contact Address:

Kerala State Commission for Protection of Child Rights

'Sree Ganesh', T. C. 14/2036, Vanross Junction

Kerala University P. O., Thiruvananthapuram - 34, Phone : 0471 - 2326603

Email: childrights.cpcr@kerala.gov.in, rte.cpcr@kerala.gov.in

Website : www.kescpcr.kerala.gov.in

Child Helpline - 1098, Crime Stopper - 1090, Nirbhaya - 1800 425 1400

Kerala Police Helpline - 0471 - 3243000/44000/45000

Online R. T. E Monitoring : www.nireekshana.org.in